Portfolio Choice with Sustainable Spending: A Model of Reaching for Yield

John Y. Campbell and Roman Sigalov

Harvard University

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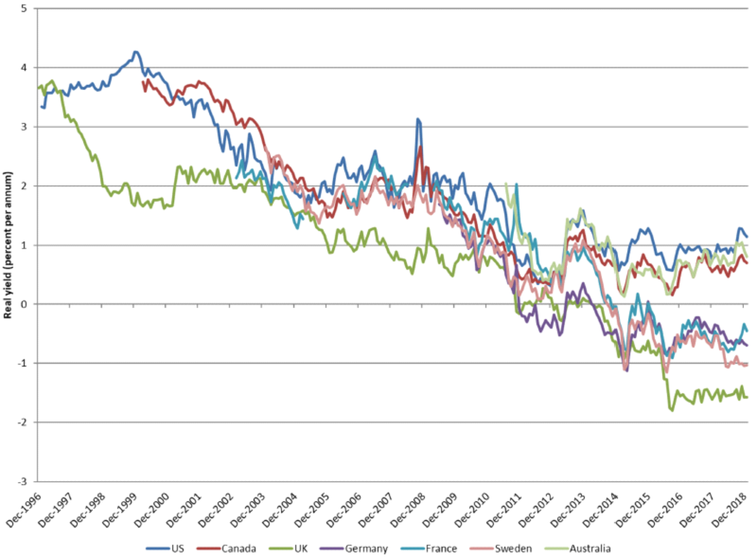
Reaching for Yield

- Prolonged decline in real interest rates: "low for long".
- If investors respond by taking more risk, they are **reaching for yield** (RFY).
- Much discussed by central bankers: RFY as a side effect or intended effect of loose monetary policy.
- But this is contrary to standard finance theory: why would it happen?

Long-Term Real Interest Rates Around the World



The Central Bankers' Discussion

Stein (2013):

> *"A prolonged period of low interest rates, of the sort we are experiencing today, can create incentives for agents to take on greater duration or credit risks, or to employ additional financial leverage, in an effort to reach for yield."*

Rajan (2013):

> *"To some extent... reach for yield is precisely one of the intended consequences of unconventional monetary policy. The hope is that as the price of risk is reduced, corporations faced with a lower cost of capital will have greater incentive to make real investments, thereby creating jobs and enhancing growth."*

The Theory Puzzle

- Standard finance theory does **not** predict RFY: risktaking depends on risk premium, risk, and risk aversion but **not** the riskfree interest rate.
- Recent literature has proposed a variety of institutional explanations for RFY:
  - Fixed nominal return target (Rajan 2013), possibly related to zero lower bound for retail deposit rates (Di Maggio and Kacperczyk 2017).
  - Low rates lower the opportunity cost of holding liquid assets (e.g. reserves) which are needed for leveraged risktaking (Drechsler, Savov, and Schnabl (2018).
  - Low rates worsen the underfunding of pension plans, which react by gambling for resurrection (Andonov, Bauer, and Cremers 2017).
  - Low rates lengthen the duration of insurance company liabilities, which react by lengthening the duration and hence the yield of their assets (Ozdagli and Wang 2019).

Our Theory

- Start with a standard Merton model of an infinitely lived investor with power utility.
- Add one constraint, realistic for endowments and sovereign wealth funds (SWFs):
  - The investor must consume the expected real return on wealth each period.
  - This makes wealth a martingale so the investor cannot plan to run down or accumulate wealth.
  - Two variants, arithmetic expected return vs. geometric expected return, differ in detail but the main results are the same.
- This **sustainable spending constraint** implies
  - RFY
  - Stronger RFY when the real interest rate is already low
  - Risktaking can respond perversely to the risk premium when the real interest rate is low
  - In a nominal variant of the model, stronger RFY when inflation is low.

The Constraint

Harvard website:

> *"The University's spending practice has to balance two competing goals: the need to fund the operating budget with a stable and predictable distribution, and the obligation to maintain the long-term value of endowment assets after accounting for inflation."*

Norges Bank Investment Management website:

> *"So that the fund benefits as many people as possible in the future too, politicians have agreed on a fiscal rule which ensures that we do not spend more than the expected return on the fund."*

RFY by a Sovereign Wealth Fund

Bloomberg News 12/1/16:

Norway's $860 billion wealth fund recommended it add about $130 billion in stocks and sell off bonds as it presented a bleak view on the returns from its investments across the globe in the decades to come.

The central bank's board, which oversees the fund, on Thursday recommended an increase in the equity share to 75% from 60%. That will raise the expected average annual real return to 2.5% percent over 10 years... compared with 2.1%... under the current setup.

The world's largest sovereign wealth fund said that it expects an annual return of only 0.25% on bonds over the next decade and that the expected equity risk premium...will be just 3% in a cautious estimate.

Broader Applicability

The sustainable spending constraint may be applicable to other investor types as well.

- Trusts with different income and principal beneficiaries, investing on behalf of the income beneficiary with a distant future date for principal disbursement.
  - Modern trust law interprets income as expected return (Sitkoff and Dukeminier 2017).
- Individuals with a behavioral reluctance to run down wealth by dissaving.
  - Our model complements the behavioral analysis by Lian, Ma, and Wang (2019) which emphasizes the role of reference rates.

Outline

- Review of the standard Merton model.
- Arithmetic sustainable spending constraint:
  - A closed-form solution with RFY.
- Geometric sustainable spending constraint:
  - Why this may be a preferable formulation.
  - Comparable RFY properties without a closed-form solution.
- Graphical analysis for intuition.
- Extensions:
  - Gifts
  - Inflation and a nominal spending constraint
  - Epstein-Zin preferences.
- A dynamic model.

Merton Model Setup

Choose both the risky share α_t and consumption c_t to

$$\max_{\alpha_t, c_t} \mathrm{E}_0 \int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \ ,$$

subject to iid returns:

$$dw_t = w_t dr_{p,t} - c_t dt,$$

$$dr_{p,t} = \alpha_t dr_t + (1 - \alpha_t) r_f dt,$$

$$dr_t = (r_f + \mu) dt + \sigma dZ_t,$$

or

$$\frac{dw_t}{w_t} = (r_f + \alpha_t \mu) dt + \alpha_t \sigma dZ_t - \frac{c_t}{w_t} dt.$$

Merton Model Solution

- Risky share:

$$\alpha_t = \alpha = \frac{\mu}{\gamma\sigma^2}.$$

  - Classic formula, no RFY!

- Consumption-wealth ratio:

$$\frac{c_t}{w_t} = \frac{\rho}{\gamma} + \frac{\gamma - 1}{\gamma}\left(r_f + \frac{1}{2\gamma}\left(\frac{\mu}{\sigma}\right)^2\right).$$

- Growth rates:

$$\frac{dw_t}{w_t} = \frac{dc_t}{c_t} = \left(\frac{r_f - \rho}{\gamma} + \frac{1+\gamma}{2\gamma^2}\left(\frac{\mu}{\sigma}\right)^2\right)dt + \frac{1}{\gamma}\left(\frac{\mu}{\sigma}\right)dZ_t.$$

  - Desired wealth and consumption growth are increasing in the riskfree rate. But what if this is not possible?

Definition of the Arithmetic Constraint

- Consumption is wealth times simple expected return:

$$c_t dt = w_t \mathrm{E}_t dr_{p,t} = w_t(r_f + \alpha\mu)dt.$$

- From the budget constraint, this implies that wealth follows a martingale with proportional volatility:

$$dw_t = w_t \alpha\sigma dZ_t.$$

- We will show that c_t / w_t is constant, so consumption also follows a martingale.

Arithmetic Maximization Problem

- Choose only the risky share to solve

$$\max_{\alpha_t} \int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \; = \int_0^\infty e^{-\rho t} \frac{\left(w_t \left(r_f + \alpha \mu\right)\right)^{1-\gamma}}{1-\gamma} dt,$$

subject to

$$dw_t = w_t \alpha \sigma dZ_t.$$

Arithmetic Solution

- The risky share is

$$\alpha = \frac{-r_f + \sqrt{K}}{\mu(1+\gamma)},$$

where

$$K = r_f^2 + 2\rho \left(\frac{1+\gamma}{\gamma}\right) \left(\frac{\mu}{\sigma}\right)^2.$$

- Standard properties:
  - Portfolio volatility $\alpha\sigma$ depends only on the Sharpe ratio μ/σ.
  - Risky share α declines with σ and γ.
- But there are nonstandard properties too!

Nonstandard Properties of the Arithmetic Model

Proposition

In the arithmetic model, the risky share α has the following properties.

1. *α is a decreasing and convex function of the riskfree rate r_f.*
2. *α is an increasing function of the rate of time preference ρ.*
3. *α is an increasing function of the risk premium μ when $r_f > 0$, and a decreasing function of μ when $r_f < 0$.*

Simple Intuition

- Lower riskfree rate or greater impatience lead the investor to want higher consumption (lower marginal utility) today relative to expected future consumption (marginal utility).
- In the standard model, this is achieved by dissaving.
- With a sustainable spending constraint, it is achieved by taking risk. This allows higher spending today, and the negative consequence (riskier consumption) is realized in the future.
- A lower risk premium has both a standard substitution effect (take less risk) and a nonstandard income effect similar to that of a lower riskfree rate.
- All the nonstandard effects get stronger as the riskfree rate declines.
  - Hence the interest in RFY today (this paper written in 2020 with 1970s technology!)
  - The nonstandard effect of the risk premium dominates when $r_f < 0$.

How Sustainable is the Arithmetic Model?

- The arithmetic model has a problem emphasized by Dybvig and Qin (2019).
- Log wealth follows the process

$$d \log(w_t) = -\frac{1}{2}\alpha^2\sigma^2 dt + \alpha\sigma dZ_t,$$

  which has a negative drift.
- The solution to this equation is

$$w_t = w_0 \exp\left\{-\frac{1}{2}\alpha^2\sigma^2 t + \sigma_c Z_t\right\}.$$

- As $t \to \infty$, since $Z_t/t \to 0$ almost surely, we have $w_t \to 0$ almost surely; and with constant c_t/w_t, $c_t \to 0$ almost surely.
- Wealth and consumption are lognormal so constant expectations result from a vanishingly small number of paths with very high values; almost all paths have values that approach zero.
  - How "sustainable" is this?

Definition of the Geometric Constraint

- Define portfolio value without subtracting consumption:

$$\frac{dV_t}{V_t} \equiv (r_f + \alpha\mu)dt + \alpha\sigma dZ_t.$$

- Consumption equals wealth times expected log return:

$$\begin{aligned} c_t dt &= w_t \mathrm{E}[d \log V_t] \\ &= w_t \left[r_f + \alpha\mu - \frac{1}{2}\alpha^2\sigma^2 \right] dt. \end{aligned}$$

- Then

$$dw_t = w_t \frac{1}{2}\alpha^2\sigma^2 dt + w_t \alpha\sigma dZ_t,$$

  and

$$d \log(w_t) = \alpha\sigma dZ_t.$$

  - **Log** wealth and consumption are martingales.
  - Normally distributed, so median future values equal current values.
  - This seems genuinely "sustainable".

Geometric Maximization Problem

- Choose only the risky share to solve

$$\max_{\alpha_t} \int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \ = \int_0^\infty e^{-\rho t} \frac{\left(w_t \left[r_f + \alpha\mu - \frac{1}{2}\alpha^2\sigma^2\right]\right)^{1-\gamma}}{1-\gamma} dt$$

subject to

$$dw_t = w_t \frac{1}{2}\alpha^2\sigma^2 dt + w_t \alpha\sigma dZ_t.$$

- The solution to this problem is uninformative in closed form (the root of a cubic equation), but applying the implicit function theorem to the HJB equation and first-order condition, we can derive similar nonstandard properties to those of the arithmetic model.
  - We state these properties for the empirically relevant case $\gamma > 1$.

Nonstandard Properties of the Geometric Model

Proposition

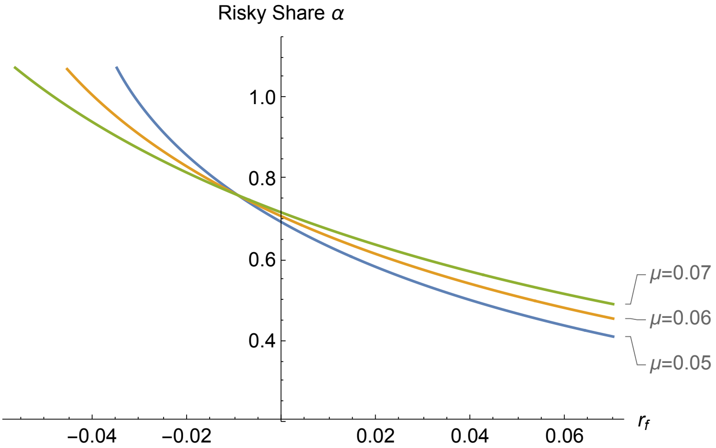
In the geometric model, the risky share α has the following properties.

1. α *is a decreasing and convex function of the riskfree rate* r_f.
2. α *is an increasing function of the rate of time preference* ρ.
3. *There exists* $r_f^* < 0$ *such that for* $r_f > r_f^*$, α *is an increasing function of the risk premium* μ *and for* $r_f < r_f^*$, α *is a decreasing function of* μ.
4. *As risk aversion* γ *approaches 1,* α *approaches the growth-optimal level* μ/σ^2 *for all values of* r_f.

Interpretation

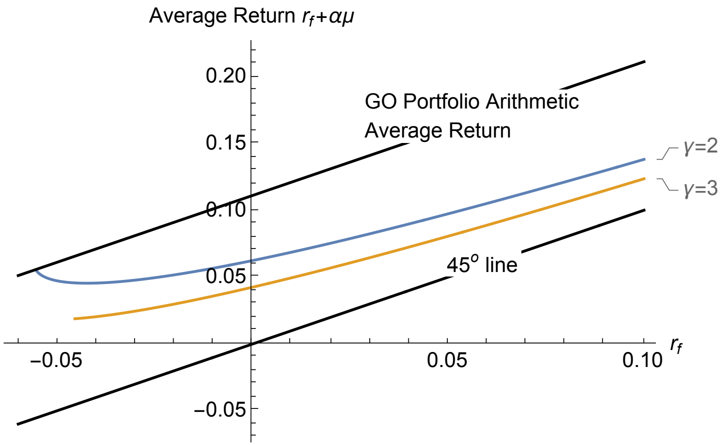
- The nonstandard effects of the riskfree rate and impatience have the same intuition as in the arithmetic model.
- The nonstandard effects get stronger as the riskfree rate declines.
- The nonstandard income effect of the risk premium dominates the standard substitution effect when the riskfree rate is sufficiently negative.
- The upper bound on risktaking is the growth-optimal portfolio with $\alpha = \mu/\sigma^2$, because this maximizes the expected log return and hence current consumption.

RFY and the Risk Premium



$\rho = 0.075$, $\gamma = 3$, $\sigma = 0.18$. Merton $\alpha = 0.62$ for $\mu = 0.06$.

Expected Returns and the Riskfree Rate



$$\rho = 0.075,\ \mu = 0.06,\ \sigma = 0.18.$$

Motivation

- To further develop intuition, we now rewrite the problem as one of choosing the initial level of consumption c_0, the mean consumption growth rate μ_c, and the volatility of consumption growth σ_c subject to constraints.
- Consider a consumption process

$$d \log c_t = \mu_c dt + \sigma_c dZ_t,$$

or equivalently

$$dc_t = c_t \left(\mu_c + \frac{1}{2}\sigma_c^2 \right) dt + c_t \sigma_c dZ_t.$$

- Then the value function for a power-utility investor is

$$v(c_0) = \left(\rho + (\gamma - 1)\mu_c - (\gamma - 1)^2 \frac{\sigma_c^2}{2} \right)^{-1} \frac{c_0^{1-\gamma}}{1-\gamma}.$$

- All the models we have considered fit into this framework.

Indifference Curves

- Rewrite the value function in the form of an indifference (iso-value) curve:

$$c_0 = \left[\left(\rho + (\gamma - 1)\mu_c - (\gamma - 1)^2 \frac{\sigma_c^2}{2} \right) (1 - \gamma) v \right]^{\frac{1}{1-\gamma}}.$$

- For a given value v, this equation determines the level of c_0 that is consistent with given μ_c and σ_c.
- Sustainable spending constraints determine μ_c and c_0 given σ_c.
- Hence, the sustainable-spending-constrained problem can be drawn on a diagram for c_0 and σ_c, analogous the classic mean-standard deviation diagram of static portfolio choice theory.

Arithmetic Indifference Curves and Constraint

- In the arithmetic model $\mu_c = -\sigma_c^2/2$: the indifference curves are

$$c_0 = \left[\left(\rho - \gamma(\gamma - 1)\frac{\sigma_c^2}{2} \right) (1 - \gamma) v \right]^{\frac{1}{1-\gamma}}.$$

  - The slope of the indifference curves is zero at $\sigma_c = 0$.
  - There is an upper bound on volatility consistent with a finite value function.

- The constraint is linear:

$$c_0 = r_f + \left(\frac{\mu}{\sigma} \right) \sigma_c,$$

where we use $\sigma_c = \alpha\sigma$.

Geometric Indifference Curves and Constraint

- In the geometric model $\mu_c = 0$: the indifference curves are

$$c_0 = \left[\left(\rho - (\gamma - 1)^2 \frac{\sigma_c^2}{2} \right) (1 - \gamma) v \right]^{\frac{1}{1-\gamma}}.$$

  - The slope of the indifference curves are again zero at $\sigma_c = 0$.
  - For common v, the geometric curve lies below the arithmetic curve except when $\sigma_c = 0$.
  - The upper bound on volatility consistent with a finite value function is greater in the geometric case.

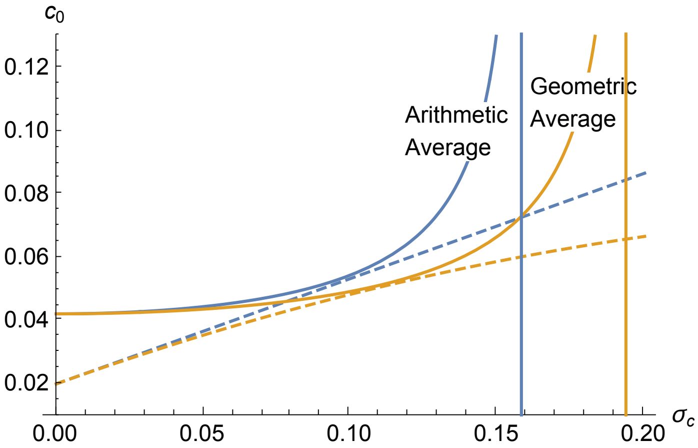
- The constraint is nonlinear:

$$c_0 = r_f + \left(\frac{\mu}{\sigma} \right) \sigma_c - \frac{1}{2} \sigma_c^2,$$

  where we use $\sigma_c = \alpha\sigma$.

- We will focus primarily on results for the geometric model.

Arithmetic and Geometric Constraints



$$\rho = 0.075,\ \gamma = 3,\ r_f = 0.02,\ \mu = 0.06,\ \sigma = 0.18.$$

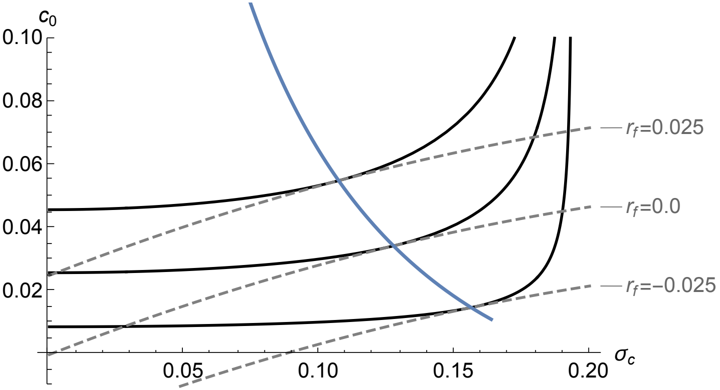
Existence of a Solution

- For a solution to exist, there must be a point where an indifference curve is tangent to the spending constraint.
- This requires that the maximum of the constraint is above zero.
  - Trivially satisfied in the arithmetic model.
  - In the geometric model, the expected return on the growth-optimal portfolio must be positive.
- Also, the constraint must reach the positive quadrant at a low enough level of volatility for the value function to be defined.
  - This can fail if the riskfree rate is negative, the Sharpe ratio is low, the rate of time preference is low, and risk aversion is high.

Graphical Intuition for RFY

- As value decreases, both the level and slope of the indifference curves decrease at any given σ_c (because value is multiplicative).
- A decrease in the riskfree rate is a parallel shift down in the constraint so it decreases value.
- But then, at the original σ_c there is no longer a tangency. The new tangency has a higher σ_c where the indifference curve is steeper and the geometric constraint is flatter.
- These effects are stronger when the riskfree rate is low, until the point is reached where the problem no longer has a solution.

Reaching for Yield

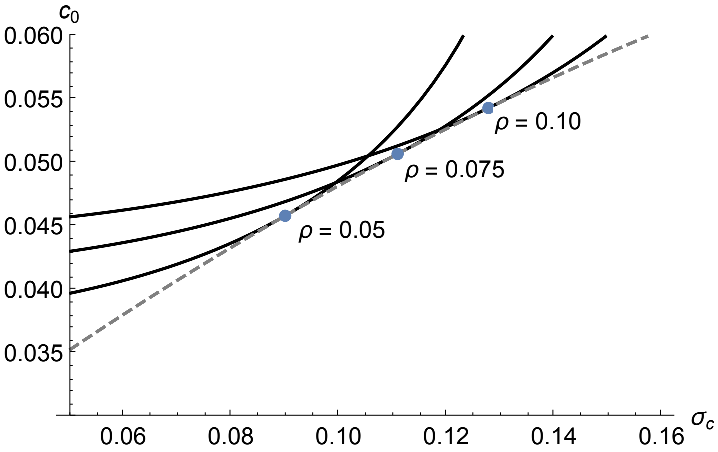


$$\rho = 0.075,\ \gamma = 3,\ \mu = 0.06,\ \sigma = 0.18.$$

Graphical Intuition for Impatience

- As the investor becomes more impatient (has a higher rate of time preference), the indifference curves become flatter at any given σ_c.
- Hence, the tangency point shifts to the right implying a higher σ_c.
- In the limit as time preference increases, the indifference curves are horizontal and the investor chooses the growth-optimal portfolio where the constraint is also horizontal.

Impatience

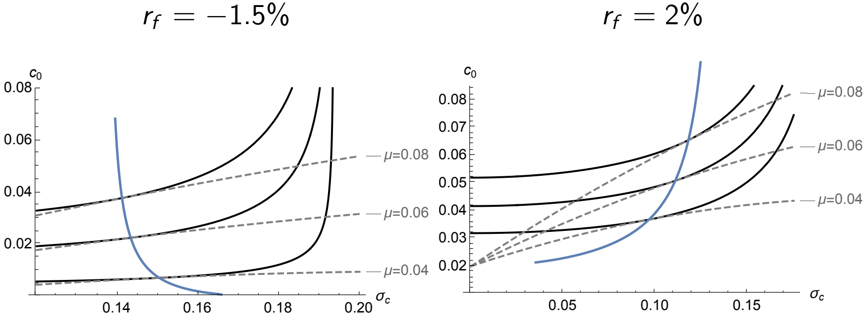


$$\gamma = 3,\ r_f = 0.02,\ \mu = 0.06,\ \sigma = 0.18.$$

Graphical Intuition for the Risk Premium Effect

- An increase in the risk premium makes the spending constraint steeper but also raises it whenever the portfolio initially takes risk.
- The former substitution effect increases risktaking, but the latter income effect reduces it.
- The income effect is stronger whenever initial risktaking is greater, and hence is stronger at low levels of the riskfree interest rate.
- If the riskfree rate is sufficiently negative, the income effect dominates.

Response to the Risk Premium



$\rho = 0.075,\ \gamma = 3,\ \sigma = 0.18.$

Current-Use and Endowment Gifts

- Consider an endowment that receives gifts, proportional to the current level of wealth (for tractability).
- Current-use gifts, arriving at rate g_u, can be spent in the period they are received.
- Endowment gifts, arriving at rate g_e, are added to wealth and spent sustainably later.
- The intertemporal budget constraint becomes

$$dw_t = w_t dr_{p,t} + w_t(g_u + g_e)dt - c_t dt,$$

- The arithmetic sustainable spending constraint becomes

$$c_t dt = w_t \left(\mathrm{E}_t dr_{p,t} + g_u\right) = w_t(r_f + g_u + \alpha\mu)dt,$$

  and the geometric constraint is similarly modified.
- Only current-use gifts enter the sustainable spending constraint, but both types of gifts enter the intertemporal budget constraint.

The Effects of Gifts

- The indifference curve becomes

$$c_0 = \left[\left(\rho + (\gamma - 1) g_e - \gamma(\gamma - 1) \frac{\sigma_c^2}{2} \right) (1 - \gamma) v \right]^{\frac{1}{1-\gamma}}.$$

- The constraint can be written as

$$c_0 = r_f + g_u + \frac{\mu}{\sigma} \sigma_c.$$

- Current-use gifts are equivalent to an increase in the riskfree interest rate.
  - Hence, they discourage risktaking (reverse RFY effect).
- Endowment gifts are equivalent to an increase in the rate of time preference.
  - Hence, they encourage risktaking because risk has current benefits and deferred costs.
- Intuitively, current-use gifts relax the sustainable spending constraint while endowment gifts tighten it.

A Constant Inflation Rate

- Consider an environment with constant inflation:

$$dp_t = p_t \pi dt.$$

- The nominal riskfree rate is

$$r_f^{\$} = r_f + \pi,$$

and the nominal return on the risky asset is

$$dr_t^{\$} = (r_f + \pi + \mu)dt + \sigma dZ_t.$$

Nominal Arithmetic Spending Constraint

- Suppose the investor is constrained to spend sustainably in nominal terms:

$$c_t^{\$}dt = w_t^{\$}\mathrm{E}[dr_{p,t}^{\$}],$$

where $c_t^{\$} = c_t p_t$ and $w_t^{\$} = w_t p_t$.

- This implies

$$c_t dt = w_t(r_f + \pi + \alpha\mu)dt,$$

and real wealth shrinks at average rate π:

$$\frac{dw_t}{w_t} = -\pi dt + \alpha\sigma dZ_t.$$

Indifference Curve with a Nominal Arithmetic Constraint

- The indifference curve becomes

$$c_0 = \left[\left(\rho - (\gamma - 1)\pi - \gamma(\gamma - 1)\frac{\sigma_c^2}{2} \right) (1-\gamma) v \right]^{\frac{1}{1-\gamma}}.$$

- The constraint can be written as

$$c_0 = r_f + \pi + \left(\frac{\mu}{\sigma} \right) \sigma_c.$$

- Comparing with the previous real model, inflation has two effects:
  - It subtracts $(\gamma - 1)\pi$ from the rate of time preference in the indifference condition.
  - It adds π to the riskfree interest rate in the constraint.
- Both effects reduce risktaking, so if the riskfree real rate is constant a decline in inflation leads to RFY by nominally constrained investors.
- If the nominal rate is held constant, the first effect still operates so again a decline in inflation leads to RFY by nominally constrained investors.

Effects of Inflation

- The analysis of a geometric sustainable spending constraint is similar.
  - Again, inflation subtracts from the rate of time preference and adds to the riskfree interest rate.
  - Low inflation leads to RFY by nominally constrained investors whether we hold the real riskfree rate or the nominal riskfree rate constant.
- These results help to explain why RFY is more of a concern today than in the 1970s.
  - Currently, both the real interest rate and inflation are low so our model predicts strong RFY by both real and nominally constrained investors.
  - In the 1970s, the real interest rate was low but inflation was even higher, so the nominal interest rate was high and our model does not predict RFY by nominally constrained investors.

Epstein-Zin Preferences

- Following Duffie and Epstein (1992), we model Epstein-Zin preferences in continuous time as

$$V_t = \mathrm{E}_t \int_t^{\infty} f(c_s, V_s) ds,$$

where the aggregator function

$$f(c, V) = \frac{1}{1 - \psi^{-1}} \left(\frac{\rho c^{1-\psi^{-1}}}{((1-\gamma)V)^{\frac{\gamma - \psi^{-1}}{1-\gamma}}} - \rho(1-\gamma)V \right).$$

  - Risk aversion is γ, the elasticity of intertemporal substitution (EIS) is ψ, and the power utility restriction $\gamma = \psi^{-1}$ is relaxed.

Solution with Epstein-Zin Preferences

- The closed-form solution for the arithmetic case becomes

$$\alpha = \frac{-r_f + \sqrt{L}}{\mu(1+\psi^{-1})},$$

where

$$L = r_f^2 + 2\rho \left(\frac{1+\psi^{-1}}{\gamma} \right) \left(\frac{\mu}{\sigma} \right)^2 .$$

- This is the same as before, replacing some instances of γ with ψ^{-1}.
- It has the same nonstandard properties as before, but also the risky share is increasing in ψ. It approaches zero as ψ approaches zero, regardless of the level of risk aversion.
- The geometric case is similarly modified, has the same nonstandard properties as before, and also has a risky share increasing in ψ.

From Comparative Statics to Dynamics

- So far we have done comparative static analysis of models with iid returns, comparing equilibria with permanently different levels of riskfree interest rates.
- Do our results carry over to a dynamic model in which the riskfree interest rate moves over time?
  - In such a model, do we get RFY as the riskfree interest rate declines?
- In a dynamic model, the sustainable spending constraint implies that wealth (or log wealth) is a martingale, but consumption is not.
  - However, consumption will be close to a martingale if the riskfree rate is persistent. Accordingly we assume this.

Dynamic Model Setup

- The investor solves

$$\max_{\alpha_t} E_0 \int_0^{\infty} e^{-\rho t} u(c_t) dt$$

subject to $c_t = w_t \left(r_{ft} + \alpha_t \mu - \frac{1}{2}\alpha_t^2 \sigma^2 \right)$

$$\begin{pmatrix} dw_t \\ dr_{ft} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} w_t \alpha_t^2 \sigma^2 \\ \phi(r_{ft}) \end{pmatrix} + \begin{pmatrix} w_t \alpha_t \sigma & 0 \\ \nu r_{ft} \eta & \nu r_{ft} \sqrt{1-\eta^2} \end{pmatrix} \begin{pmatrix} dZ_{1t} \\ dZ_{2t} \end{pmatrix}$$

- We set

$$\phi(r_{ft}) = \frac{1}{2} \nu^2 r_{ft}$$

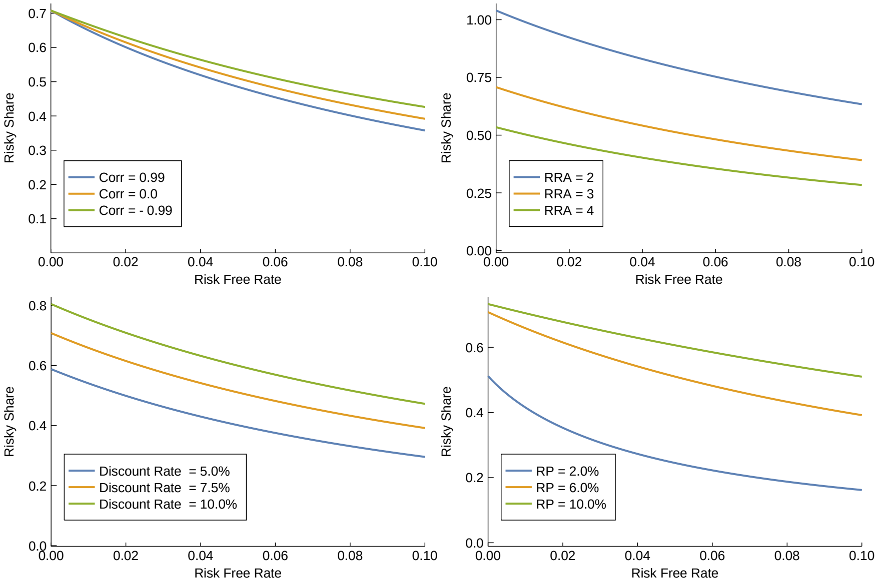
in order to make $\log(r_{ft})$ a martingale.

  - This implies that log consumption is a martingale for an agent who takes zero risk, but not exactly a martingale for a risktaking agent.

Dynamic Model Results

- This model again delivers RFY, and a more impatient investor takes more risk.
- The risk premium always has a positive effect on risktaking because the riskfree rate is always positive in this model.
  - However, the effect of the risk premium on risktaking is smaller when the riskfree rate is low.
- The model also predicts an intertemporal hedging demand for the risky asset as in Merton (1993) and Campbell and Viceira (2001).

Dynamic Model



Conclusion

- Classical finance theory separates risktaking from intertemporal choice.
- Our model breaks this separation using a sustainable spending constraint: investors take risk as a way to increase current consumption at the cost of more volatile future consumption.
  - The model predicts RFY, stronger when the riskfree rate is low, and more risktaking by impatient investors.
- Rampini and Viswanathan (2010, 2013, 2019) similarly break the separation using models of collateral constraints for firms or borrowing constraints for households.
  - Constrained firms will not put up collateral today, and constrained households will not pay insurance premia today, to manage their future risk exposures.
- The classical result is not as robust as finance theorists have supposed.