A Q-Theory of Inequality

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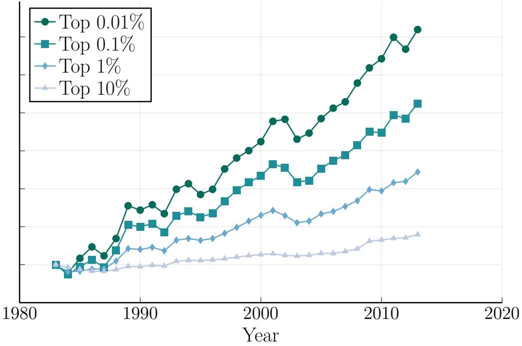
Interest Rate and Pareto Inequality

- Recent rise in top wealth inequality: "fattening" of the right tail



- Classical view: tail of the wealth distribution increases with r

  Wold and Whittle (1957) ...Piketty and Zucman (2015)

Our Paper

1. **We argue that low r can increase top wealth inequality**
    - While low r decreases the growth rate of existing fortunes...
    - ...it *increases* the growth rate of new fortunes

Our Paper

2. **Sufficient statistic to quantify the effect of r on the Pareto tail**

- Agents start as entrepreneurs with concentrated portfolio

  $\rightarrow$ Transition to rentiers as firms mature

- Sufficient statistic depends on equity payout yield + leverage of firms owned by entrepreneurs *reaching the top*

Our Paper

3. We measure the sufficient statistic in the data

- We collect new data on the wealth trajectory of top entrepreneurs
- A 5% decline in r can explain 3/4 of the rise in top wealth inequality

Stylized Model

Environment

- Continuum of infinitely-lived agents. Population grows at rate η
- New agents are born “entrepreneurs” and endowed with a tree
- Trees require investment by“rentiers” to grow
- Eventually, trees blossom and entrepreneurs become rentiers themselves

Environment

- Trees have initial size of one

  ... requires continuous flow of investment i

  ... grows at rate g

  ... blossoms with hazard rate δ, giving a one-time dividend equal to its size

- Formally, the instantaneous cash-flow $\mathrm{d}D_t$ is given by

$$\mathrm{d}D_t = \begin{cases} -ie^{gt}\,\mathrm{d}t & \text{conditional on growing} \\ e^{gt} & \text{if blossoms} \\ 0 & \text{afterwards.} \end{cases}$$

Market Value of a Tree

- Denote r the interest rate
- Denote q the market-to-book ratio of the tree:

$$r = \underbrace{-\frac{i}{q} + g}_{\substack{\text{return conditional}\\\text{on growing}}} + \delta \underbrace{\left(\frac{1}{q} - 1\right)}_{\substack{\text{return conditional}\\\text{on blossoming}}}$$

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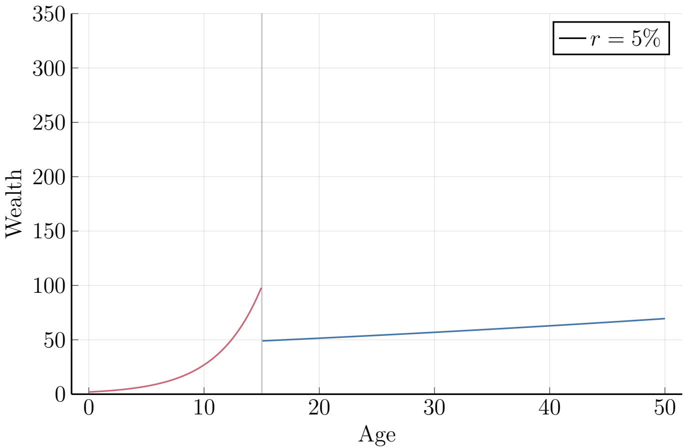
$\Rightarrow$ A decline in r has two effects:

1. lowers the ex-ante return (left-hand side)
2. changes the distribution of ex-post returns (right-hand side)

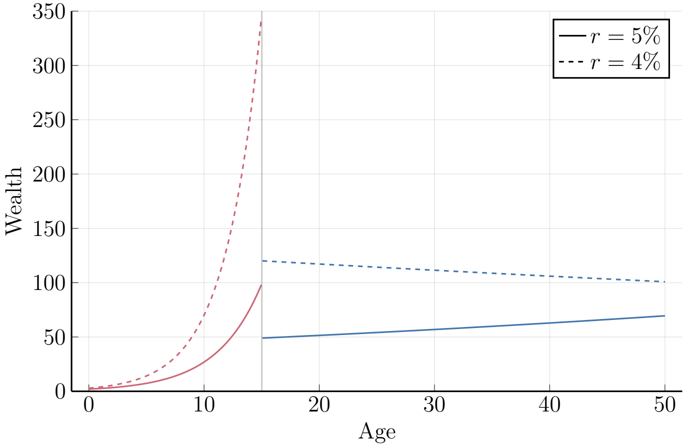
Wealth Accumulation

- Life-cycle
  1. Agent starts as an entrepreneur: invests in their tree until it blossoms
  2. Agent then becomes a rentier: invests in a diversified portfolio of trees
- Agents have log-utility with subjective discount factor ρ
- The law of motion of individual wealth is

$$\frac{\mathrm{d}W_t}{W_t} = \begin{cases} \left(-\frac{i}{q} + g - \rho\right)\mathrm{d}t & \text{when entrepreneurs} \\ \frac{1}{q} - 1 & \text{when tree blossoms} \\ \left(r - \rho\right)\mathrm{d}t & \text{when rentiers} \end{cases}$$



Realized wealth path of an agent with a tree blossoming after 15 years



While lower r decreases the growth rate of rentiers...

...it *increases* the growth rate of successful entrepreneurs

Pareto Inequality

- **Def:** A distribution has a Pareto tail if $\mathbb{P}(W > w) \sim Cw^{-\frac{1}{\theta}}$ as $w \to +\infty$

  θ measures the thickness of the tail

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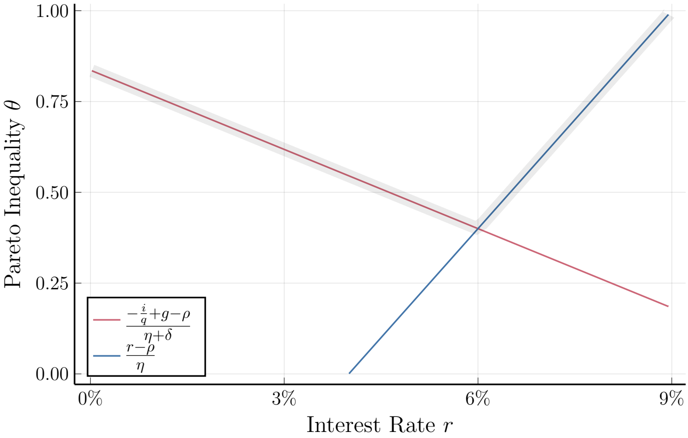
  θ measures the thickness of the tail

- **Prop**: Pareto inequality θ is given by:

$$\theta = \max\left(\frac{-\frac{i}{q} + g - \rho}{\eta + \delta}, \frac{r - \rho}{\eta}\right)$$



Pareto Inequality θ
1.00
0.75
0.50
0.25
0.00
0%
3%
6%
9%
Interest Rate r
$\frac{-\frac{i}{q}+g-\rho}{\eta+\delta}$
$\frac{r-\rho}{\eta}$

Sufficient Statistic

General Model

- Agents are born “entrepreneurs” with a firm of size K_0
- Firms have aK technology with convex adjustment costs $i(g)$
- TFP $a \in \{a_1, \ldots, a_s\}$ follows a Markov Chain with transition matrix $\mathcal{T}$
- At rate δ, entrepreneurs disinvest in their firms and become rentiers

Firm Problem

- Firm growth g is optimally chosen to maximum firm's value $V_s(K)$:

$$rV_s(K) = \max_g \left\{(a_s - i(g))K + V_s'(K)gK + (\mathcal{T}V)_s(K)\right\}$$

- Homogeneity gives $V_s(K) = q_s K$:

$$\begin{aligned} i'(g_s) &= q_s \\ r &= \frac{a_s - i(g_s)}{q_s} + g_s + \frac{(\mathcal{T}q)_s}{q_s} \end{aligned}$$

Book Wealth

- Define book wealth of an entrepreneur: $B_t = W_t / q_s$
- Evolution of book wealth:

$$\frac{dB_t}{B_t} = \underbrace{\left(\frac{a_s - i(g_s)}{q_s} + g_s - \rho \right)}_{\mu_s} \mathrm{d}t$$

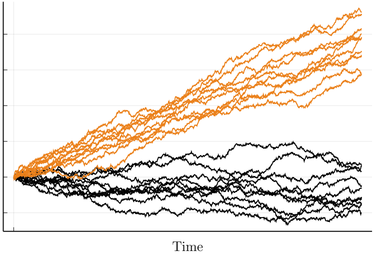
- The effect of r on the growth rate of book wealth is

$$\partial_r \mu_s = \underbrace{\frac{a_s - i(g_s)}{q_s}}_{\text{payout yield}} \underbrace{\left| \partial_r \log q_s \right|}_{\text{duration}} + \underbrace{\left(1 - \frac{i'(g_s)}{q_s} \right) \partial_r g_s}_{=0} .$$

Pareto Inequality

- We characterize analytically the effect of r on Pareto inequality

$$\frac{\partial_r \theta}{\theta} = \mathbb{E}\left[\frac{\partial_r \mu_s}{\mu_s} \middle| \text{reaching the top}\right]$$



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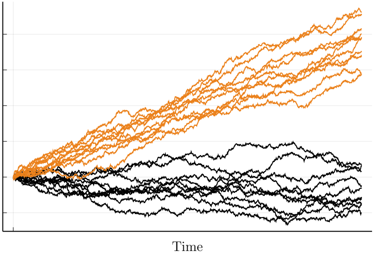

- We obtain a sufficient statistic:

$$\frac{\partial_r \theta}{\theta} = \mathbb{E}\left[\frac{\text{payout yield} \times \text{duration}}{\text{growth rate}} \middle| \text{reaching the top}\right]$$

Leverage

We extend the model to consider firms with leverage

- Lower r has two distinct effects on μ_S:
  1. Increases valuation, which reduces dilution
  2. Increases the cash-flow of equity-holders at the expense of debt-holders

- Sufficient statistic becomes:

$$\frac{\partial_r \theta}{\theta} = \mathbb{E}\left[\frac{\text{equity payout yield} \times \text{duration} - \text{debt-to-equity}}{\text{growth rate}} \middle| \text{reaching the top}\right]$$

Empirics

Estimating the Sufficient Statistic

1. Collect data on the wealth trajectory of the top 50 entrepreneurs
   (i) Equity payout yield (CRSP post-IPO, SEC-1 pre-IPO)
   (ii) Debt-to-equity
   (iii) Growth rate

2. Estimate the sufficient statistic as

$$\frac{\widehat{\partial_r \theta}}{\theta} = \frac{1}{N} \sum_{1=1}^{N} \frac{\text{equity payout yield}_i \times \text{duration} - \text{debt-to-equity}_i}{\text{growth rate}_i}.$$

Example of Mark Zuckerberg

Table 1: Capitalization Table for Facebook

	Founding Date	Angel Round	Series A	...	IPO
Founders	100%	90%	72%	...	28%
Employees	0%	0%	5%	...	32%
Outside Investors	0%	10%	23%	...	40%

(i) Equity payout yield $\approx$ -11%

(ii) Debt-to-Equity $\approx$ 5% (virtually no debt)

(iii) Growth Rate of Wealth $\approx$ 108% (0.1 million to 41 billions in 12 years)

Results

- We collect data on the top 50 U.S. entrepreneurs

	Average	Percentiles				
		Min	p25	p50	p75	Max
Equity Payout Yield	-2.2%	- 16%	-4.9%	0.1%	0.8%	3.7%
Debt to Equity	39%	3%	19%	39%	39%	194%
Growth Rate	30%	15%	20%	22%	32%	108%

- We consider an average duration of 30 years (Gormsen-Lazarus, 2019)

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$\Rightarrow$ Putting everything together, we obtain

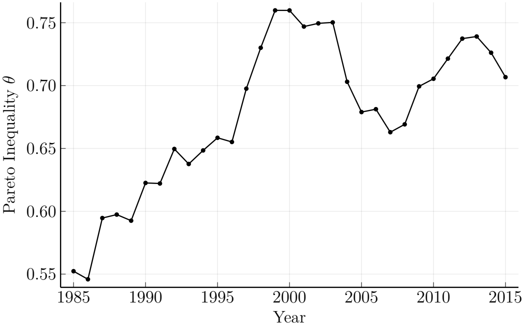
$$\frac{\widehat{\partial_r \theta}}{\theta} = -3.6$$

Decline of r and Rise in Pareto Inequality

- Discount rates have decreased by ≈ 5% since 1980s
- This can explain a decline in Pareto inequality by:

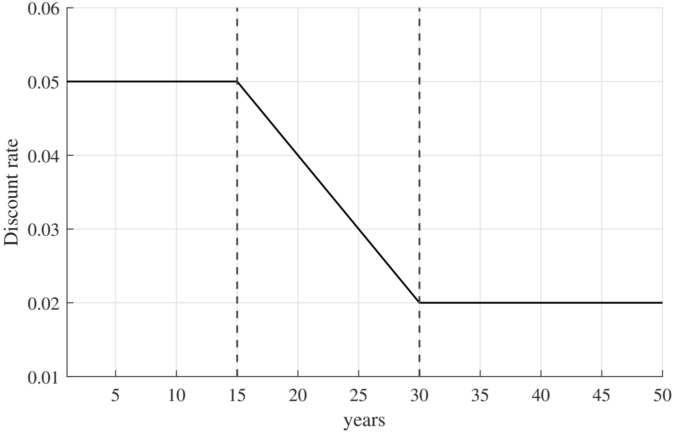
$$\frac{\widehat{\partial_r \theta}}{\theta} \times \Delta r = -3.6 \times -5\% \approx 18\%$$

- This accounts for 3/4 of the rise in Pareto inequality since 1980s

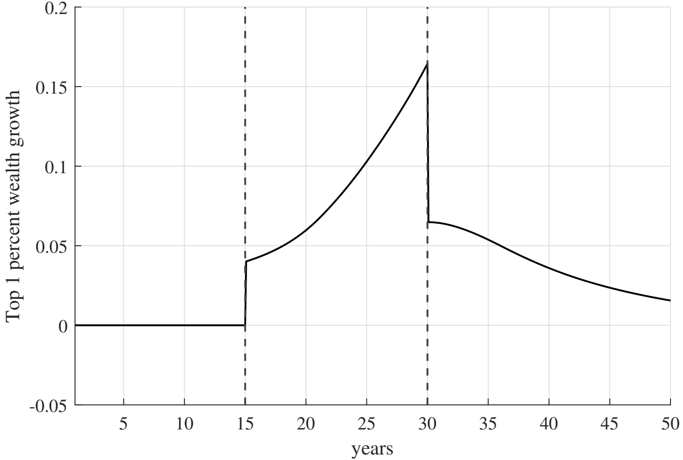


Transition Dynamics

Interest Rate Path



Growth rate of wealth in the top 1%.

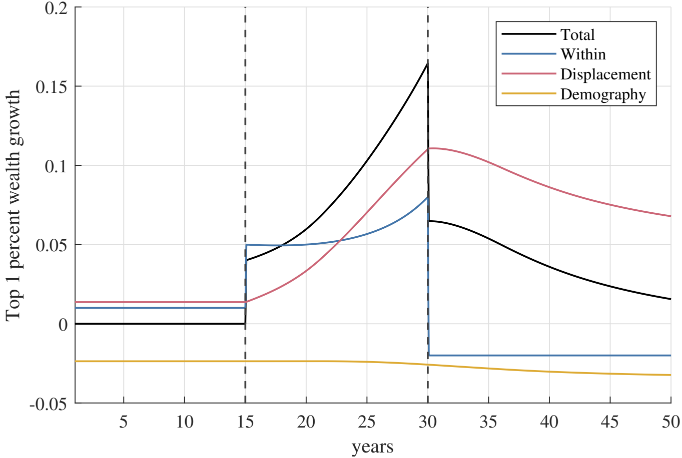


Decomposition

$$\frac{\overline{W}_{T_2,2} - \overline{W}_{T_1,1}}{\overline{W}_{T_1,1}} = \underbrace{\frac{\overline{W}_{T_1,2} - \overline{W}_{T_1,1}}{\overline{W}_{T_1,1}}}_{\text{Within}} + \underbrace{s_E \frac{\overline{W}_{E,2} - q_2}{\overline{W}_{T_1,1}} + s_X \frac{q_2 - \overline{W}_{X,2}}{\overline{W}_{T,1}}}_{\text{Displacement}} + \underbrace{\eta \frac{q_2 - \frac{\overline{W}_{T_1,2}}{\overline{W}_{T_1,1}} \overline{W}_{T,1}}{\overline{W}_{T_1,1}}}_{\text{Population Growth}}$$

- T_t set of people in the top 1% at time t
- E: set of people that enter the top percentile
- X: set of people that exit the top percentile
- q_2: wealth of the last person in the top percentile

Growth rate of wealth in the top 1%.



Conclusion

- We overturn a classical result: lower r can increase Pareto inequality
- 5% decline in discount rates $\rightarrow$ 3/4 of the rise in Pareto inequality
- Magnitude depends on characteristics of the economy

  Esp. high in the U.S. due to scalable firms + developed financial markets