How the Wealth Was Won: Factor Shares as Market Fundamentals

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Sharp Rise in Equity Values in Post-War Period

- Stock market risen sharply in post-war era, driven mostly *last 30 years*.

	Average Annual Growth		
Subsample	**Market Equity**	**Output**	**Earnings**
1989:Q1 - 2017:Q4	7.5%	2.6%	5.1%
1966:Q1 - 1988:Q4	1.6%	3.9%	1.8%

Notes: Variables for the U.S. corporate sector. Annualized growth rates for the specified sample, in real terms, deflated by the implicit price deflator for nonfinancial corporate sector output (net value added).

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- Upshot? Widening **chasm** between **stock market** and **broader economy**.

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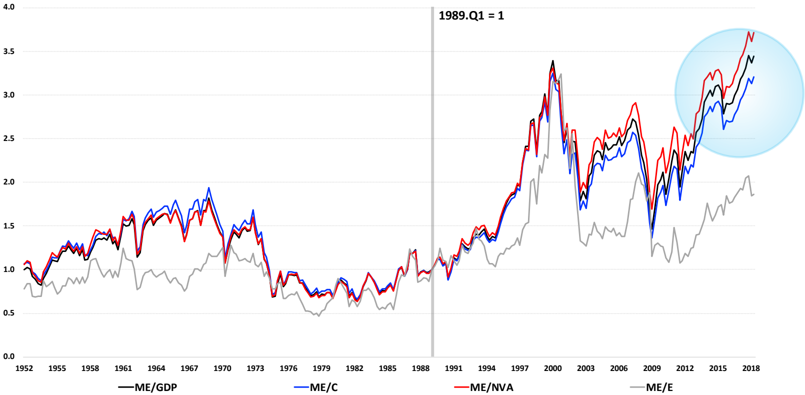
Stock Market v.s Broader Economy

- ME= Total value of market equity of the U.S. corp sector.

Notes: **ME:** Corporate Sector Stock Value. **E:** Corporate Business After-Tax Profits. **GDP & C:** Current Dollars GDP and personal consumption expenditures. **NVA:** Net Value Added of Corporate Sector. The sample spans the period 1952:Q1-2018:Q2.

Stock Market v.s Broader Economy

- ME relative to 3 different measures of agg. economic activity is at or near post-war high.



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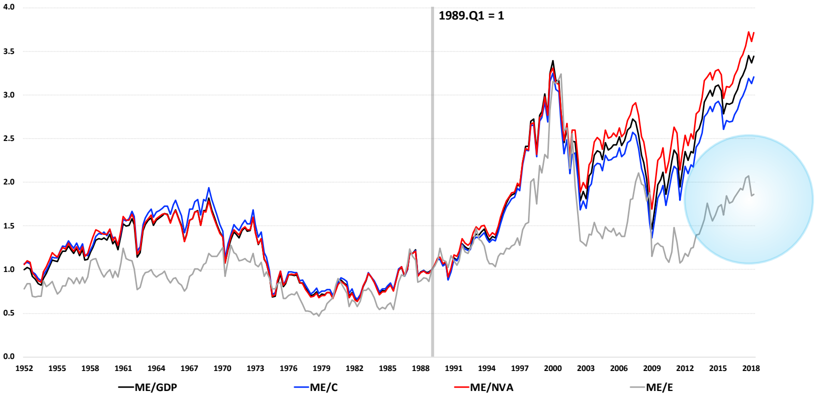
- Notably, ME/E not near post-war high.



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Macro-Finance Trends

- Textbook economics teaches us: stock market and economy should contain a *common trend* (goes back to at least Klien and Kosobud ’61).
- Very **factors** that **boost economy** are also **key to rising equity values** over long periods.
- Figure 1 suggests basic **tenet of macroeconomic** theory not borne out by data.
- *What is responsible for sharply rising equity values over post-war period?*

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  3. **Economic growth**: Could still be key to market's rise over post-war period, even if last 30 years have been a striking exception.
- On the potential importance of 1: wide & persistent swings in *profit share* of output cause long-lasting deviations between corp. cash-flows and the value of what the sector produces.
  - CS after-tax profit share of output ranges from **less than 8% to nearly 20%** over our sample.

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- Identification of **mutually uncorrelated** components + **loglinear** model => precisely decompose 100% of market's observed growth into **distinct component sources** in the model.
- Apply model to the U.S. corporate sector (CS) over period 1952:Q1-2017:Q4.

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- **Economic growth** contributed **25%** since 1989, and **54%** over full 65 year sample.
- From 1952-1988, **growth** accounted for **111%**, but these **37 years** created *less than a third* of the wealth generated over **29 years since 1989**.

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- Implication: **high returns** to holding equity in post-war period have been in large part attributable to *long sequence of redistributive shocks* that reallocated rewards to shareholders.

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- Estimate: ≈ 2.9 **percentage points** of post-war avg. annual *log* return on **equity in excess of short term interest rate** attributable to this string of favorable shocks.

Related Literature

- **Drivers of real level of stock market**: Few studies. Lettau & Ludvigson '13, and Greenwald, Lettau, Ludvigson (GLL) '14.
- This paper replaces GLL, differs substantively from both. Neither study did formal estimation of asset pricing model. GLL model is less flexible, less general.
- **Limited participation**: Mankiw '86; Mankiw, Zeldes '91; Vissing-Jorgensen '02; Ait-Sahalia et. al., '04, Guvenen '09. In contrast to this, GLL, Lettau et. al., '19 and this paper: investors are **concerned about redistributive shocks** that have opposite effects on labor and capital.
- **Decline in labor share**: Karabarounis, Neiman '13, Lansing '13.
- **Negative correlation returns human wealth and stock market**: Lustig, Van Nieuwerburgh '08; Lettau, Ludvigson '09; Chen et. al., '14.
- **Macro-finance trends**: Farhi and Gourio '18; Corhay, Kung, Schmid '18.

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- Workers consume labor income (no assets). Shareholders akin to **wealthy hous. or inst. investor** finances consump. from assets.
- **Aggregate domestic output**:

$$Y_t = A_t N_t^{\alpha} K_t^{1-\alpha}$$

A_t mean zero TFP; N_t labor endowment (hours $\times$ prod. factor).

  - Workers inelastically supply labor; hours fixed, normalized to 1.
  - K_t grows deterministically at gross rate $G \equiv 1 + g \Rightarrow K_t = K_0 G^t$.
  - Labor productivity grows: $N_t = G^t$.

The Model: Earnings Accounting

- Fraction τ_t of Y_t devoted to **taxes & interest & other**.
- Remaining $1 - \tau_t$ divided between labor compensation and domestic after-tax profits, E_t^D.
- **Total earnings** E_t also includes retained earnings from firms' foreign subsidiaries, $E_t^F = F_t Y$.

$$E_t \equiv \mathbf{S}_t Y_t = \left(\mathbf{S}_t^D Z_t + F_t \right) Y_t$$

$Z_t \equiv 1 - \tau_t$; S_t^D **dom. profit share** and F_t **for. earnings share** of Y.

- Labor compensation

$$W_t N_t = \left(1 - S_t^D \right) Z_t Y_t,$$

- $E_t / Y_t \equiv \mathbf{S}_t$ "**earnings share**"and $\left(1 - \mathbf{S}_t^D \right)$ "**dom. labor share**".
- $\mathbf{S}_t$ moves inversely with $1 - \mathbf{S}_t^D$ and τ_t, and positively with F_t.

Factors Share Shock

- Variable $\mathbf{S_t}$ modeled as exogenous *factors share shock*.
- Reduced form way of capturing changes may occur, for any reason, in allocation of rewards to shareholders.
- Labor share component $1 - S_t^D$ is quantitatively large. Possible sources of variation include:
  1. **Industry concentration** structure alters labor intensivity of production
  2. **Bargaining power** of US workers (international competition, prevalence of unions, off-shoring)
  3. **Technological factors** alter substitutability of labor for capital.
- Earnings from **overseas affiliates** and **taxes & interest & other** make up the remaining components of the factor share process S_t.

The Model: Corporate Cashflows

- **Cash payments to shareholders** = *net payout* ("cashflows") differs from E_t by **net new investment**.
- Firm reinvests fixed fraction ωY_t each period =>

$$\underbrace{C_t}_{\text{cashflows}} = E_t - \omega Y_t = (S_t - \omega)\, Y_t.$$

- Reinvestment needed to achieve long-term growth in Y_t at rate g.
- Simple way to capture the empirical fact that firms in agg retain part of E for reinvestment, and that this required investment depends on Y rather than S.

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- In equilibrium, agg. SH consumption = agg. **net payout** C_t.

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- In equilibrium, agg. SH consumption = agg. **net payout** C_t.
- Distinguished from **representative household** model in which the agent consumes per capita **aggregate consumption**.

The Model: SDF

- IMRS of *shareholder* consumption is the **SDF** and takes the form:

$$\begin{aligned} M_{t+1} &= \beta_t \left(\frac{C_{t+1}}{C_t}\right)^{-x_t}, \qquad \beta_t \equiv \frac{\exp(-\delta_t)}{\exp(d_t)} \\ \ln M_{t+1} &= -\mathbf{1}'\delta_t - d_t - x_t \Delta \ln C_{t+1} \end{aligned}$$

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- x_t drives **price of risk** in SDF; affects risk premia.
- Since an SDF always reflects both preferences and beliefs, interpret a decrease in x_t as *either* a decrease in **effective risk aversion** or decrease in **pessimism**.
- Time varying β_t essential for obtaining stable **risk-free rate** along with volatile equity premium.

Loglinear Model: Output and Earnings

- Work with loglinear approximation solved analytically. (E_t/Y_t) could go above 1, but never does so (0% of time in 10,000 period simulation) b/c estimated S_t process $>$ 14 std away from unity in steady state.
- Lowercase letters denote log variables. All shocks are modeled as Gaussian, independent over time, and mutually uncorrelated.

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- **TFP and Output growth**:

$$\Delta a_{t+1} = \varepsilon_{a,t+1}, \quad \Delta y_{t+1} = g + \varepsilon_{a,t+1}, \quad \varepsilon_{a,t+1} \sim N\, i.i.d. \left(0, \sigma_a^2\right).$$

- **Earnings**: Since $E_t = S_t Y_t$, earnings growth

$$\Delta e_t = \Delta s_t + \Delta y_t.$$

Loglinear Model: Payout

- **Payout**: $C_t = (S_t - \omega) Y_t$, log-linearize around $c_t - y_t = \overline{cy}$.
- **Approximate expression** for payout ratio:

$$c_t - y_t = \overline{cy} + \xi s_t,$$

where $\xi = \frac{\overline{S}}{\overline{S} - \omega}$ and $\overline{S}$ is the average value of S_t.

- Generalize cash-flow growth equation

$$\Delta c_t = \boldsymbol{\xi} \Delta \mathbf{s}_t + \Delta y_t. \qquad (1)$$

Vector $\mathbf{s}_t$ to model **components of** s_t as a mixture of multiple stochastic processes.

- Data plainly suggest the presence of **lower and higher frequency components in** s_t: modeled as $s_{LF,t}$ and $s_{HF,t}$.

$$s_t = s_{LF,t} + s_{HF,t}$$

- From (1), we have $\mathbf{s}_t = (s_{LFt}, s_{HFt})'$ and $\boldsymbol{\xi}' = (\xi, \xi)$.

Loglinear Model: Dynamics of Cashflows

- Specify dynamics of Δc_t as

$$\begin{aligned} \Delta c_{t+1} &= \boldsymbol{\xi}' \Delta \mathbf{s}_{t+1} + \Delta y_{t+1} & \\ \mathbf{s}_{t+1} &= (\mathbf{I} - \boldsymbol{\Phi}_s)\bar{\mathbf{s}} + \boldsymbol{\Phi}_s \mathbf{s}_t + \boldsymbol{\varepsilon}_{s,t+1}, & \boldsymbol{\varepsilon}_{s,t+1} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_s) \\ \Delta \mathbf{s}_{t+1} &= -(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t + \boldsymbol{\varepsilon}_{s,t+1}, & \widetilde{\mathbf{s}}_t \equiv \mathbf{s}_t - \bar{\mathbf{s}} \end{aligned}$$

- $\boldsymbol{\Phi}_s$ is a diagonal matrix with autoregressive coefficients of $s_{LF,t}$ and $s_{HF,t}$ in diagonal entries.

- $\boldsymbol{\Sigma}_s$ is a diagonal covariance matrix.

Loglinear Model: Risk Free Rate

- **Risk-free rate of return** known with certainty at t:

$$R_{f,t+1} \equiv \left(\mathbb{E}_t\left[M_{t+1}\right]\right)^{-1}, \quad \beta_t \equiv \frac{\exp\left(-\delta_t\right)}{\exp\left(d_t\right)}.$$

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- Model $\delta_t = \mathbf{1}'\boldsymbol{\delta}_t$, where $\boldsymbol{\delta}_t = \left(\delta_{LFt}, \delta_{HFt}\right)'$ and

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- Parameter d_t is a compensating factor chosen to ensure

$$r_{f,t} = -\ln \mathbb{E}_t \exp\left(m_{t+1}\right) = \mathbf{1}'\delta_t.$$

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- Data on short rates suggests **low- and higher-frequency components**.
- Model $\delta_t = \mathbf{1}'\boldsymbol{\delta}_t$, where $\boldsymbol{\delta}_t = \left(\delta_{LFt}, \delta_{HFt}\right)'$ and

$$\begin{aligned} m_{t+1} &\equiv \ln M_{t+1} = -\mathbf{1}'\boldsymbol{\delta}_t - d_t - x_t \Delta c_{t+1} \\ \boldsymbol{\delta}_{t+1} &= (\boldsymbol{I} - \boldsymbol{\Phi}_\delta)\bar{\boldsymbol{\delta}} + \boldsymbol{\Phi}_\delta \boldsymbol{\delta}_t + \boldsymbol{\varepsilon}_{\delta,t+1}, \ \boldsymbol{\varepsilon}_{\delta,t+1} \sim N(0, \boldsymbol{\Sigma}_\delta), \end{aligned}$$

- Parameter d_t is a compensating factor chosen to ensure

$$r_{f,t} = -\ln \mathbb{E}_t \exp\left(m_{t+1}\right) = \mathbf{1}'\delta_t.$$

- Gaussian shocks, the SDF is **conditionally lognormal**:

$$r_{f,t+1} = \mathbf{1}'\delta_t + d_t + x_t\left[g - \boldsymbol{\xi}'(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t\right] - \frac{1}{2}x_t^2\left(\sigma_a^2 + \boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi}\right)$$

$$d_t = -x_t\left[g - \boldsymbol{\xi}'(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t\right] + \frac{1}{2}x_t^2\left(\sigma_a^2 + \boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi}\right)$$

Risk Price Dynamics

- Assume the **Price of risk x_t** follows:

$$x_t = \underbrace{\mathbf{1}'\mathbf{x}_{\perp,t}}_{x_{\perp,t}} + \boldsymbol{\lambda}'\widetilde{\mathbf{s}}_t$$

$$\mathbf{x}_{\perp,t+1} = \left(I - \boldsymbol{\Phi}_{x_\perp}\right)\overline{\mathbf{x}}_\perp + \boldsymbol{\Phi}_{x_\perp}\mathbf{x}_{\perp,t} + \boldsymbol{\varepsilon}_{x_\perp,t+1}, \quad \boldsymbol{\varepsilon}_{x_\perp,t+1} \sim N\, i.i.d.\left(0, \boldsymbol{\Sigma}_{x_\perp}\right).$$

  where $\mathbf{x}_{\perp,t} = (x_{\perp,LF,t}, x_{\perp,HF,t})'$ a vector of low- and high-frequency components, $\boldsymbol{\lambda} = (\lambda, \lambda)'$.

- $x_{\perp,t}$ is a component orthogonal to economic state.

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  where $\mathbf{x}_{\perp,t} = (x_{\perp,LF,t}, x_{\perp,HF,t})'$ a vector of low- and high-frequency components, $\boldsymbol{\lambda} = (\lambda, \lambda)'$.

- $x_{\perp,t}$ is a component orthogonal to economic state.
- $\lambda \neq 0$ permits correlation between earnings share and risk premia, potentially because the willingness to bear risk rises as profit shares increase.
- Data: $\ln(E/Y)$ *positively* correlated with $\ln(P/E)$ (also CRSP $\ln(P/D)$), esp over longer horizons. Impossible to explain this fact with $\lambda = 0$, since a transitory increase in s_t would lead to a *decline* in $\ln(P/E)$; a perm. increase would have no effect. Figure
- λ is freely estimated with flat priors and could in principle be 0.

Loglinear Model Solution

$pc_t \equiv \ln\left(\frac{P_t}{C_t}\right)$. Guess and verify the solution:

$$pc_t = A_0 + \mathbf{A}_s'\widetilde{\mathbf{s}}_t + \mathbf{A}_r'\widetilde{\delta}_t + \mathbf{A}_{x_\perp}'\widetilde{\mathbf{x}}_{\perp,t}$$

$$\begin{aligned}
\mathbf{A}_s' &= -\left[\boldsymbol{\xi}'(I-\Phi_s) + \left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right)\boldsymbol{\lambda}'\right]\left((I-\kappa_1\Phi_s) + (\kappa_1\Sigma_s\boldsymbol{\xi})\,\boldsymbol{\lambda}'\right)^{-1} \\
\mathbf{A}_{x_\perp}' &= -\left[\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_g^2\right) + \kappa_1'\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\mathbf{A}_s\right](I-\kappa_1\boldsymbol{\Phi}_{x_\perp})^{-1} \\
\mathbf{A}_\delta' &= -\mathbf{1}'\,(\mathbf{I}-\kappa_1\boldsymbol{\Phi}_\delta)^{-1}
\end{aligned}$$

- Sign of coefficients:
  - $\mathbf{A}_\delta'$ and $A_{x_\perp}' < 0$: $\uparrow$ risk-free rate or in price of risk increases the rate at which future cash payments **discounted**.
  - For $\boldsymbol{\lambda} = 0$, $\mathbf{A}_s' < 0$ since $\boldsymbol{\Phi}_s < 1$. Equity values rise proportionally less than c_t in anticipation of eventual **mean-reversion in payout**.
  - For $\boldsymbol{\lambda} < 0$, $\mathbf{A}_s'$ could be > 0, depending on magnitude of $\boldsymbol{\lambda}$.

Loglinear Model Solution

- Model solution implies **log equity premium**:

$$\mathbb{E}_t[r_{t+1}] - r_{f,t} = \left[\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_a^2\right) + \kappa_1'\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\mathbf{A}_s\right]\left(\mathbf{1}'\widetilde{\mathbf{x}}_{\perp,t} + \boldsymbol{\lambda}'\widetilde{\mathbf{s}}_t\right) - \frac{1}{2}\mathbb{V}_t(r_{t+1}),$$

$$\mathbb{V}_t(r_{t+1}) = \kappa_1^2\left(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{A}_s + \mathbf{A}_{x_\perp}'\boldsymbol{\Sigma}_x\mathbf{A}_{x_\perp} + \mathbf{A}_\delta'\boldsymbol{\Sigma}_\delta\mathbf{A}_\delta\right) + \boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_a^2 + 2\kappa_1'\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\mathbf{A}_s,$$

- **Homoskedastic shocks**: $\mathbb{V}_t$ constant, but risk premium varies with $\mathbf{x}_{\perp,\mathbf{t}}$, possibly $\widetilde{\mathbf{s}}_\mathbf{t}$ if $\lambda \neq 0$.

Estimation and Data

- **Primitive parameters $\boldsymbol{\theta} =$**

  $$\left(\boldsymbol{\xi}, g, \sigma_a^2, \operatorname{diag}\left(\boldsymbol{\Phi}_s\right)', \operatorname{diag}\left(\boldsymbol{\Phi}_{x_\perp}\right)', \operatorname{diag}\left(\boldsymbol{\Phi}_\delta\right)', \operatorname{diag}\left(\boldsymbol{\Sigma}_s\right)', \operatorname{diag}\left(\boldsymbol{\Sigma}_{x_\perp}\right)', \operatorname{diag}\left(\boldsymbol{\Sigma}_\delta\right)', \bar{s}, \bar{\delta}, \bar{x}_\perp\right)'$$

- **Two groups**
  - Small number calibrated (discussed below).
  - Remaining parameters freely estimated.
- **Estimation of Parameters**: Bayesian methods with *flat priors*.
- **Estimation of Latent States**: Model linear in logs so can use **Kalman filter**.

Estimation and Data

- Confront model with observations **1952:Q1-2017:Q4** on:

  1. Log output growth Δy_t
  2. Log earnings share $e_t - y_t \equiv ey_t = \ln(S_t^D Z_t + F_t)$
  3. Log risk-free rate $r_{f,t}$
  4. Equity-to-output ratio $p_t - y_t \equiv py_t$
  5. Risk premium implied by SVIX (Martin '17): rp_t

- Martin '17 uses options data compute a lower bound on equity premium and argues its approximately tight. Because of our mixture process, our risk-premium can account for high-freq component implied by options, as well as lower-freq component implied by valuation ratios.

- **Risk-free rate** 3-Mo T-bill minus fitted $\widehat{\pi}_t$ from regression on lagged π_t.

- **Observations on 1, 2, and 4 are for U.S. corporate sector**.

  - Need y_t, ey_t, py_t etc., to be measured for *same sector* of economy. Otherwise subject to confounding compositional effects.

Estimation and Data

- Forgoing variables are related to $\boldsymbol{\theta}$ and **latent states**:

$$
\begin{aligned}
s_t &= \mathbf{1}'\mathbf{s}_t \\
r_{ft} &= \mathbf{1}'\delta_t \\
py_t &= pc_t + cy_t \\
&= \overline{py} + \left(\mathbf{A}_s' + \boldsymbol{\xi}'\right)\widetilde{\mathbf{s}}_t + \mathbf{A}_\delta'\widetilde{\delta}_t + \mathbf{A}_{x_\perp}\widetilde{\mathbf{x}}_{\perp,t} \\
rp_t &= \left[\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_a^2\right) + \kappa_1\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\mathbf{A}_s\right]\left(\mathbf{1}'\widetilde{\mathbf{x}}_{\perp,t} + \lambda'\widetilde{\mathbf{s}}_t\right) - \frac{1}{2}\mathbb{V}_t(r_{t+1}) \\
\Delta y_t &= g + \Delta\tilde{y}_t
\end{aligned}
$$

where $cy_t \equiv c_t - y_t$ and $\overline{py} \equiv A_0 + \bar{c} + \xi'\bar{\mathbf{s}}$.

Estimation and Data

- **State space form**:

$$\begin{aligned} \mathcal{Y}_t &= \mathbf{H}_t' \boldsymbol{\beta}_t + \mathbf{b}_t && (2) \\ \boldsymbol{\beta}_t &= \boldsymbol{F}\boldsymbol{\beta}_{t-1} + \varepsilon_t, && (3) \end{aligned}$$

- **Observation equation**: $\mathcal{Y}_t \equiv \left(s_t, r_{ft}, py_t, rp_t, \Delta y_t\right)'$.

- **Latent states**: $\boldsymbol{\beta}_t \equiv \left(\tilde{s}_{LF,t}, \tilde{s}_{HF,t}, \tilde{\delta}_{LF,t}, \tilde{\delta}_{HF,t}, \tilde{x}_{\perp,LF,t}, \tilde{x}_{\perp,HF,t}, \Delta\tilde{y}_t\right)'$, where

$$\varepsilon_t = \left(\varepsilon_{s,LF,t}, \varepsilon_{s,HF,t}, \varepsilon_{\delta,LF,t}, \varepsilon_{\delta,HF,t}, \varepsilon_{x_\perp,LF,t}, \varepsilon_{x_\perp,HF,t}, \varepsilon_{a,t}\right)'.$$

and $\mathbf{F}$, $\mathbf{H}_t'$, and $\mathbf{b}_t$ are matrices of primitive parameters.

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  and $\mathbf{F}$, $\mathbf{H}_t'$, and $\mathbf{b}_t$ are matrices of primitive parameters.

- **Kalman filter** gives *smoothed* estimates of latent states $\boldsymbol{\beta}_{t|T}$.

- **Measurement error effectively zero** in (2) due to flexible loglinear model and use of 7 latent states to match 5 observables.

Estimation and Data

- **Posterior of θ**: Obtained by computing likelihood using Kalman filter and combining with priors.
- **Flat priors**: posterior coincides with likelihood, posterior mode coincides with MLE estimate.
- **Parameter uncertainty**: Characterized using a RWMH algorithm.
- **Latent state uncertainty** Characterized using simulation smoother of Durbin and Koopman (2002).
- **Error bands** therefore reflect both parameter and latent state uncertainty.

Estimation and Data

- **Four parameters are calibrated**: $g, \bar{\delta}, \bar{s}, \xi$.
- **Means of observable series** $\Delta y_t, r_{f,t}, ey_t$: conservative approach of fixing them at sample means.
- **Payout-earnings growth relation** ξ

$$\Delta c_t = \boldsymbol{\xi}' \Delta \mathbf{s}_t + \Delta y_t.$$

  - $\boldsymbol{\xi} = (\xi, \xi)'$, where $\xi \equiv \frac{\bar{S}}{\bar{S}-\omega}$ pinned down by data since $\frac{\bar{C}}{\bar{Y}} = \bar{S}_t - \omega => \xi = 2.19$.
  - We confirm in our results that $\xi = 2.19$ yields average growth and volatility of payouts close to those observed in data.

Parameter and Latent State Estimates

Parameter Estimates

- Effective mean risk price modest reflecting volatility cash payments to shareholders.

Variable	Symbol	Mode	5%	Median	95%
Risk Price Mean	$\bar{x}_{\perp}$	4.0460	3.3619	4.5315	6.5421
Risk Price (HF) Pers.	$\phi_{x_{\perp},HF}$	0.6705	0.5337	0.6916	0.8074
Risk Price (HF) Vol.	$\sigma_{x_{\perp},HF}$	1.5370	1.2031	1.8191	2.9421
Risk Price (LF) Pers.	$\phi_{x_{\perp},LF}$	0.9864	0.9781	0.9855	0.9915
Risk Price (LF) Vol.	$\sigma_{x_{\perp},LF}$	0.4933	0.3525	0.5841	0.9693
Risk-Free (HF) Pers.	$\phi_{\delta,HF}$	0.5639	0.1590	0.6630	0.8849
Risk-Free (HF) Vol.	$\sigma_{\delta,HF}$	0.0012	0.0002	0.0011	0.0019
Risk-Free (LF) Pers.	$\phi_{\delta,LF}$	0.9267	0.8739	0.9147	0.9655
Risk-Free (LF) Vol.	$\sigma_{\delta,LF}$	0.0015	0.0004	0.0016	0.0020
Factor Share (HF) Pers.	$\phi_{s,HF}$	0.8787	0.7917	0.8735	0.9176
Factor Share (HF) Vol.	$\sigma_{s,HF}$	0.0534	0.0298	0.0520	0.0576
Factor Share (LF) Pers.	$\phi_{s,LF}$	0.9848	0.9363	0.9834	0.9966
Factor Share (LF) Vol.	$\sigma_{s,LF}$	0.0162	0.0074	0.0175	0.0456
Productivity Vol.	σ_a	0.0152	0.0143	0.0153	0.0165
Risk Loading, Factor Share	λ	-7.9304	-10.5362	-7.1975	-3.4726

The sample spans the period 1952:Q1-2017:Q4.

Parameter Estimates

- Short rates: $\phi_{\delta,LF} = 0.93 => $ substantial declines *recently* in $r_{f,t}$ do not rationalize anything near a permanent shift.

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Parameter Estimates

- Factors share: $\phi_{s,LF} = 0.984$ estimated to be more persistent.

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Parameter Estimates

- Risk loading: .01 unit $\uparrow$ in $\ln S$ $\approx$1% $\uparrow$ in S around mean $=>$ -0.08 decrease in x_t.

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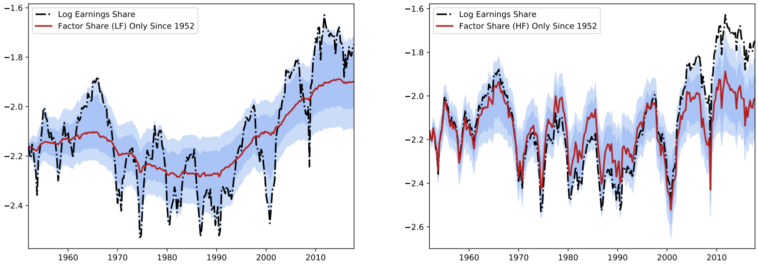
Latent States: Earnings Share

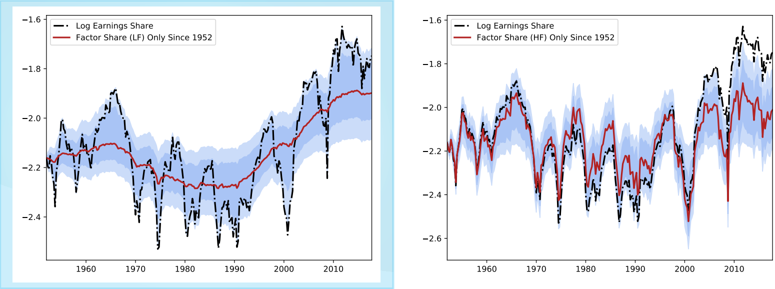
- Sum of high- and low- freq components always adds up to the observed series, without error.



The figure exhibits the observed earnings share series along with the model-implied variation in the series attributable to the latent factor share components. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Latent States: Earnings Share

- $s_{LF,t}$ captures *longer term trend* in ey_t.



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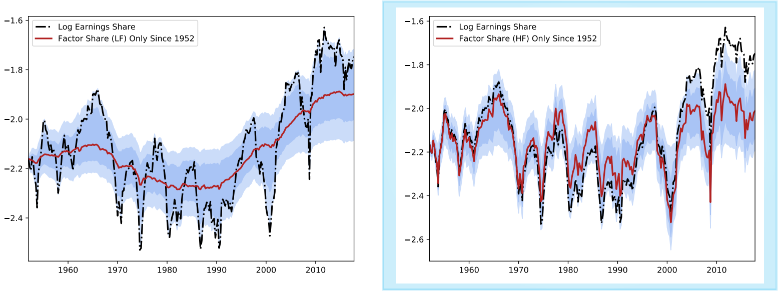
Latent States: Earnings Share

- $s_{HF,t}$ captures **transitory variation** in ey_t.



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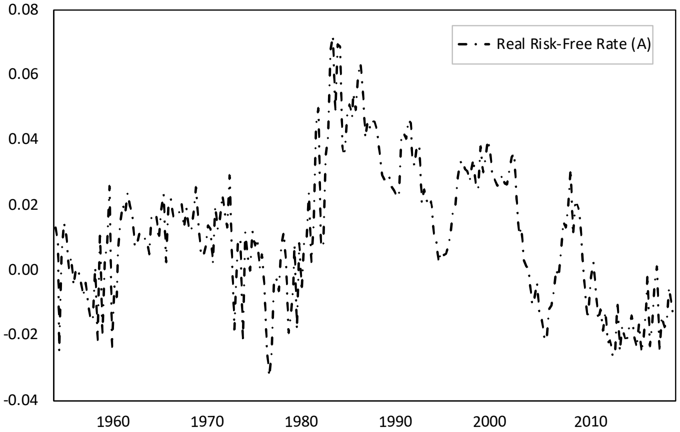
Risk-Free Rate Over Time

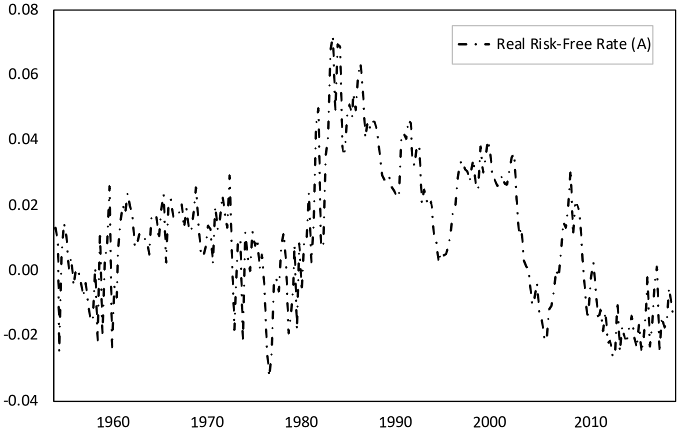
- Real rates **low** in 1950s & late 1970s, **high** during Volcker disinflation and after, **low** post-financial crisis.



The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation. The sample spans the period 1952:Q1-2017:Q4.

Risk-Free Rate Over Time

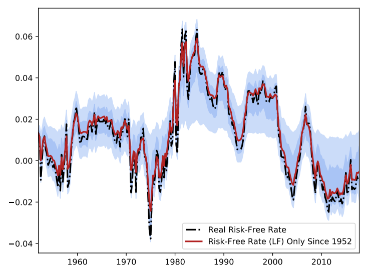
- Although rates are low today, they are **not unusually low** by historical standards.

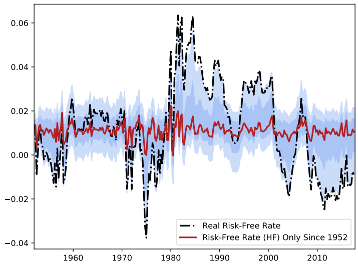


The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation. The sample spans the period 1952:Q1-2017:Q4.

Latent States: Risk-Free Rate

- **Low-high-low** pattern of $r_{f,t}$ well captured by $\delta_{LF,t}$

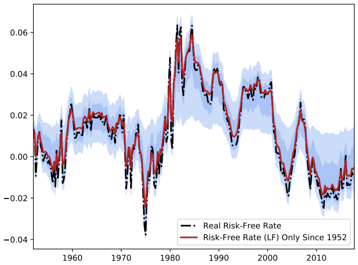


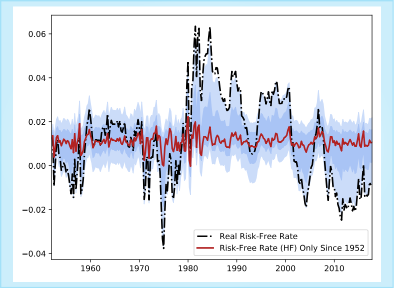


The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation and interest rates. The figure exhibits the observed risk-free rate series along with the model-implied variation in the series attributable to the latent risk-free rate components. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Latent States: Risk-Free Rate

- Component $\delta_{HF,t}$ picks up transitory variation in $r_{f,t}$.

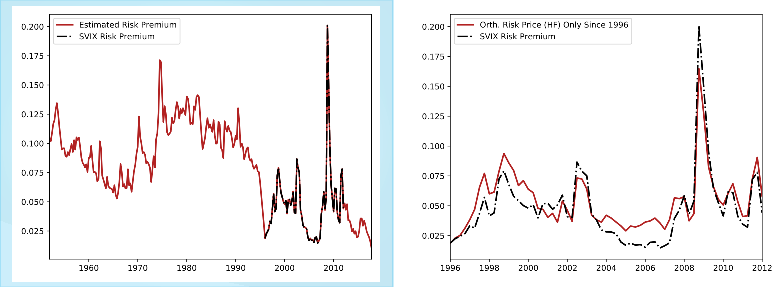




The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation and interest rates. The figure exhibits the observed risk-free rate series along with the model-implied variation in the series attributable to the latent risk-free rate components. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Latent States: Risk Premium

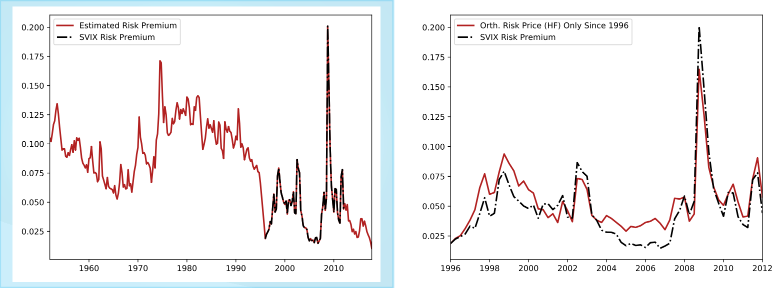
- Left panel is estimated risk premium, along with premium implied by **3mo SVIX (1996:Q1-2012:Q1)**.



Panel (a) plots the estimated risk premium over the full sample, along with the risk premium implied by the SVIX, available for the subperiod 1996:Q1-2012Q1. Panel (b) plots the component of the risk-premium driven only by $x_{\perp,HF,t}$ along with the risk premium implied by the 3-month SVIX. The label "Only Since" followed by a date describes a counterfactual path where all other components of the risk premium were held fixed from that date on. The red center line corresponds to point estimates, obtained from the Kalman smoother at the parameter mode. The sample spans the period 1952:Q1-2017:Q4.

Latent States: Risk Premium

- Except for the big spike in GFC, equity premium has been declining steadily and at record *low* at end of sample.



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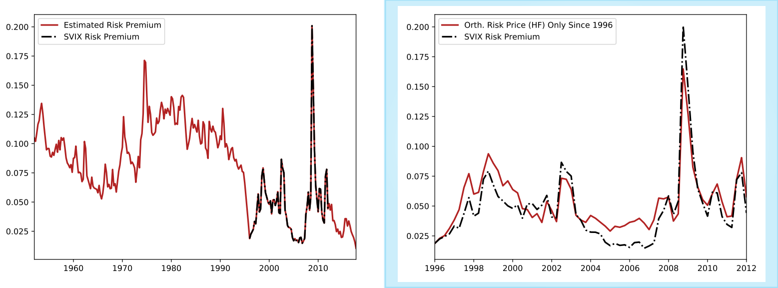
Latent States: Risk Premium

- For 1996:Q1-2012:Q1, almost all variation in premium implied by options data is ascribed to $x_{\perp,HF,t}$.



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Asset Pricing Results

Asset Pricing Moments

- "Model"numbers from **simulations**. "Fitted"numbers use **estimated latent states** obtained from fitting model to *historical data*.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
Log Risk-Free Rate	1.114	1.450	1.126	1.932	1.129	1.929
Log Excess Return	3.738	17.499	6.560	16.785	7.389	16.436
Log Price-Payout Ratio	3.778	0.404	3.410	0.376	3.434	0.465
Log Earnings Growth	2.226	8.671	2.819	11.819	2.819	11.819
Log Payout Growth	2.226	18.369	3.790	23.845	4.045	33.455
Log Earnings Share Growth	0.000	8.310	0.624	10.379	0.624	10.379
Log Payout Share Growth	0.000	18.203	1.651	22.621	1.907	32.186

All statistics are computed for annual (continuously compounded) data. "Model"numbers are averages across 1000 simulations of the model of the same size as our data sample. "Fitted"numbers use the estimated latent states fitted to observed data in our historical sample. The sample spans the period 1952:Q1-2017:Q4.

Asset Pricing Moments

- Fitted moments are model's implications *conditional on observed sequence of shocks in our sample*; are therefore **directly comparable** to "Data"moments.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
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Asset Pricing Moments

- Fitted moments of Δe_t, Δey_t, and $r_{f,t}$ match exactly b/c observables.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
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Asset Pricing Moments

- Fitted moments of log R, log excess returns, and pc_t match data moments reasonably well.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
Log Risk-Free Rate	1.114	1.450	1.126	1.932	1.129	1.929
Log Excess Return	3.738	17.499	6.560	16.785	7.389	16.436
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Asset Pricing Moments

- Fitted mean of excess return understates data mean because model understates mean PO growth over the sample (not an estimation target).

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
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Asset Pricing Moments

- Fitted mean log R^{ex} (6.6%) $>$ model mean log R_{ex} (3.7%) by 2.9 perc. points, attributable to an unusual sample with a long string of factor share shocks redistributed rewards to shareholders.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
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Asset Pricing Moments

- *Fitted means* for Δe_t and Δc_t larger than *model means*.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
Log Risk-Free Rate	1.114	1.450	1.126	1.932	1.129	1.929
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Asset Pricing Moments

- "Model"**mean excess return** reflects cov. with SDF. "Fitted"mean affected by cov. with SDF but *also* reflects persistent movements in earnings, payout over the sample.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
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Asset Pricing Moments

- Estimates imply *roughly 2.9 percentage points* of the post-war mean log return on stocks in excess of a T-bill is attributable to this string of **favorable factors share shocks**, rather than to genuine **compensation for bearing risk**.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
Log Risk-Free Rate	1.114	1.450	1.126	1.932	1.129	1.929
Log Excess Return	3.738	17.499	6.560	16.785	7.389	16.436
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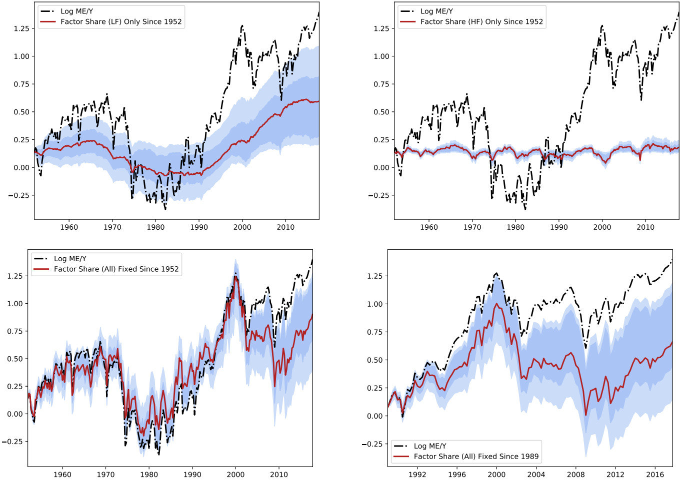
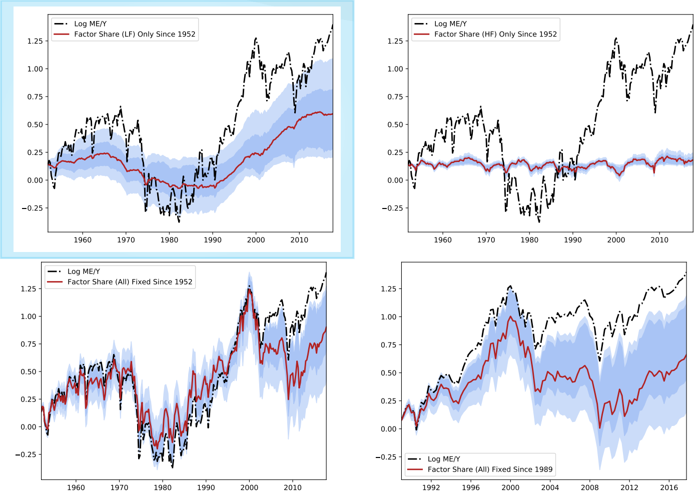
Equity Dynamics: Role of Factor Shares



The figure exhibits the observed log market equity-to-output (ME/Y) series along with the model-implied variation in the series attributable to the latent factors share components. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Equity Dynamics: Role of Factor Shares

- Longer-term swings in py_t well captured by **LF FS factor** $s_{LF,t}$.



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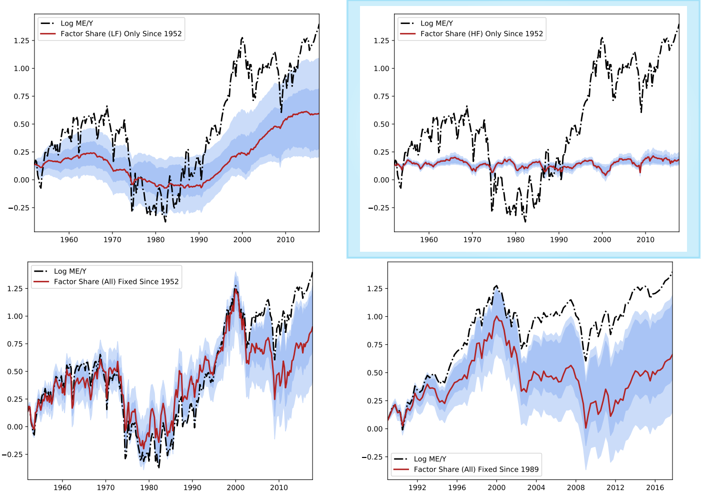
Equity Dynamics: Role of Factor Shares

- **HF FS factor** $s_{HF,t}$ captures “wiggles”.



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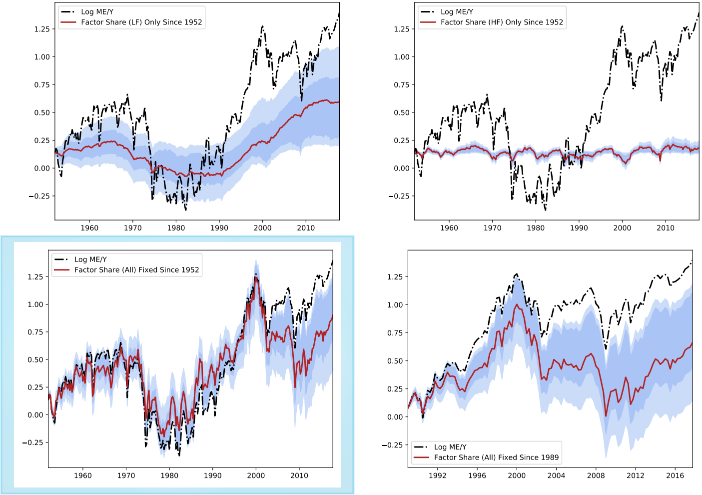
Equity Dynamics: Role of Factor Shares

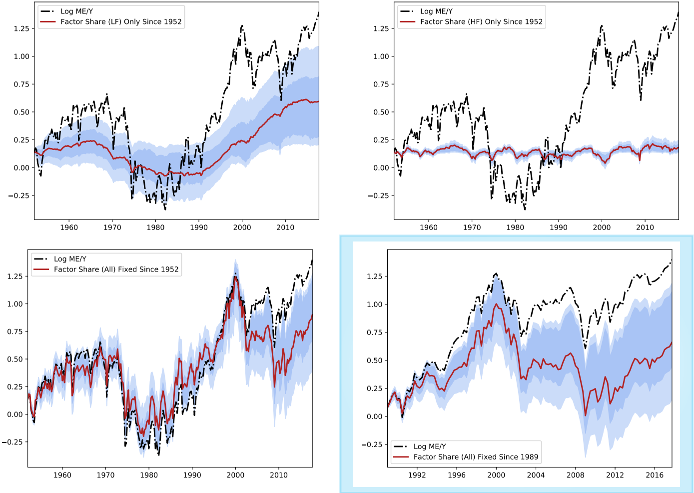
- Fix both components, model is unable to capture *any of upward trajectory* since 2000.



The figure exhibits the observed log market equity-to-output (ME/Y) series along with the model-implied variation in the series attributable to the latent factors share components. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

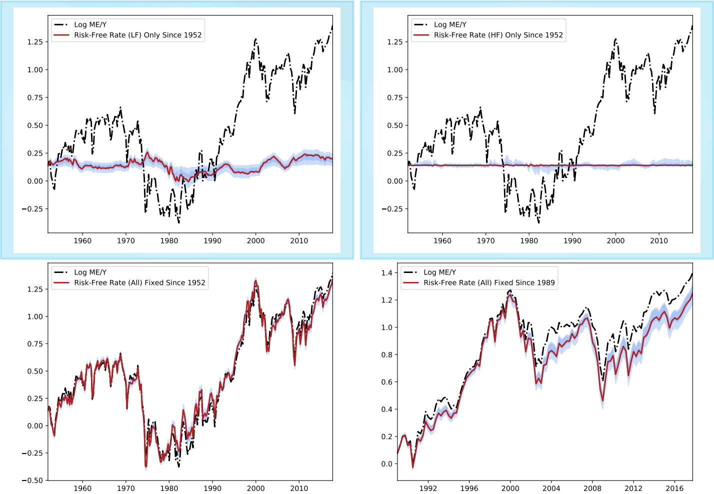
Equity Dynamics: Role of Factor Shares

- If s_t fixed at its value in 1989 **only small part of the upward trend since 1989** in py_t can be explained.



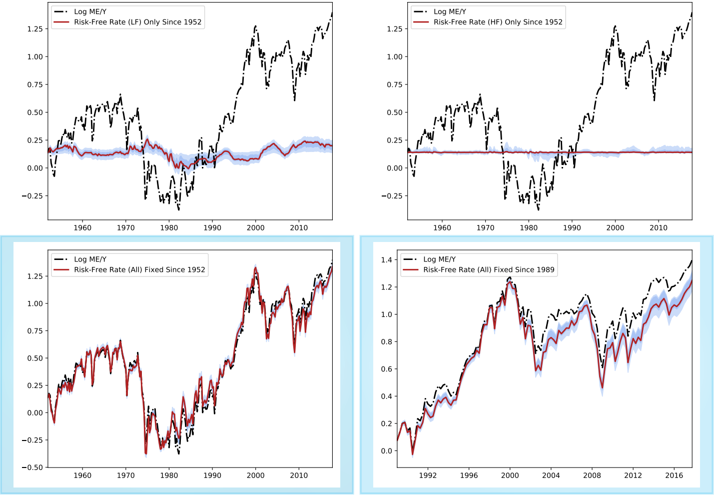
The figure exhibits the observed log market equity-to-output (ME/Y) series along with the model-implied variation in the series attributable to the latent factors share components. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

- Negligible role for either latent component in driving py_t.



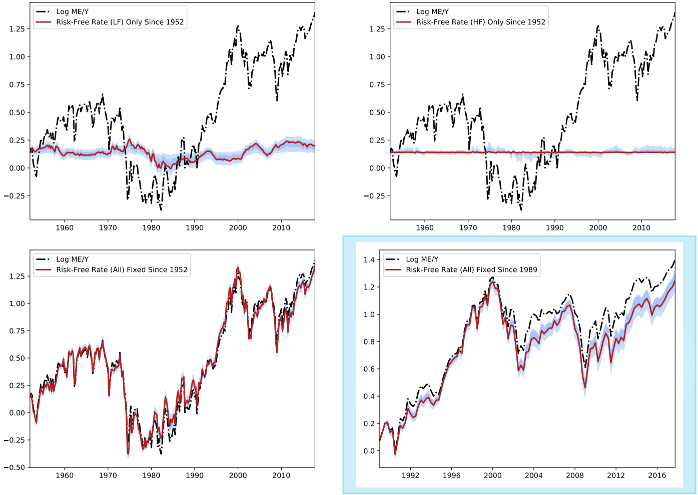
Equity Dynamics: Role of Risk-free Rate

- Shutting down either LF or HF component does little to model's ability to match **trend movements** in py_t.



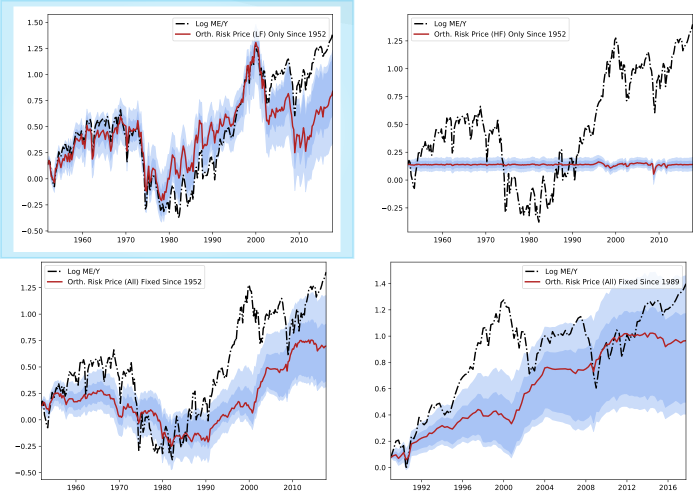
Equity Dynamics: Role of Risk-free Rate

- log ME/Y would be two-tenths log point lower with no change in $r_{f,t}$ since 1989.



Equity Dynamics: Role of Risk Price

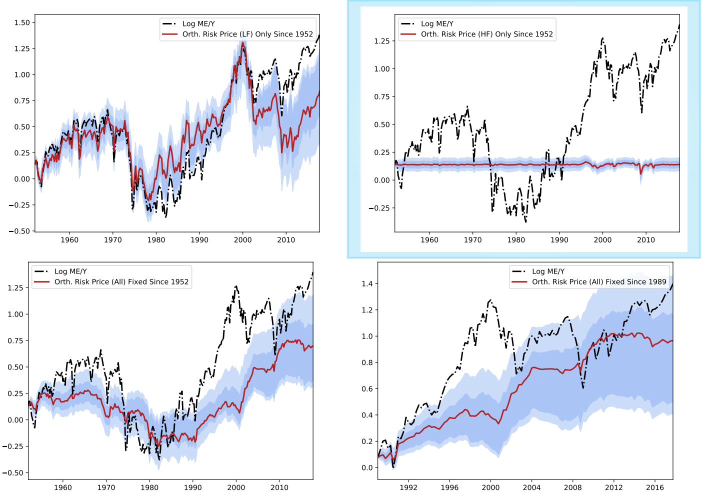
- LF risk price ($x_{\perp,LF,t}$) variation explains almost all of **transitory booms & busts**.



The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk-price component $x_{\perp,t}$. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Equity Dynamics: Role of Risk Price

- HF component explains virtually none of big swings.



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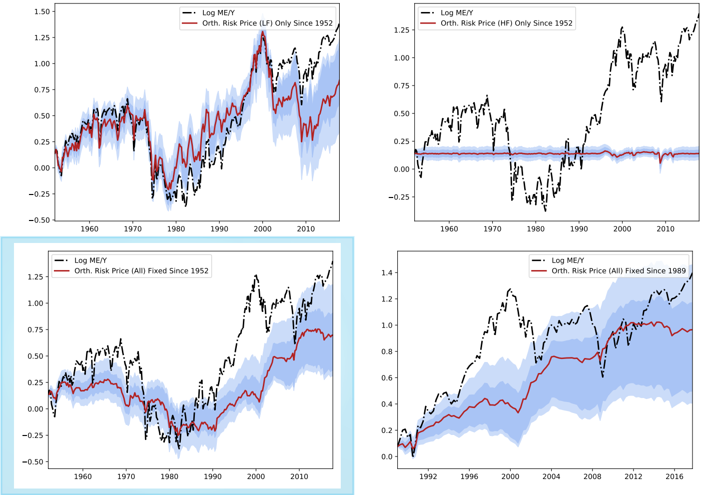
Equity Dynamics: Role of Risk Price

- Fix risk price (all) in 1952 shows $x_{\perp,t}$ explains some, but not nearly all, of rise in py_t.



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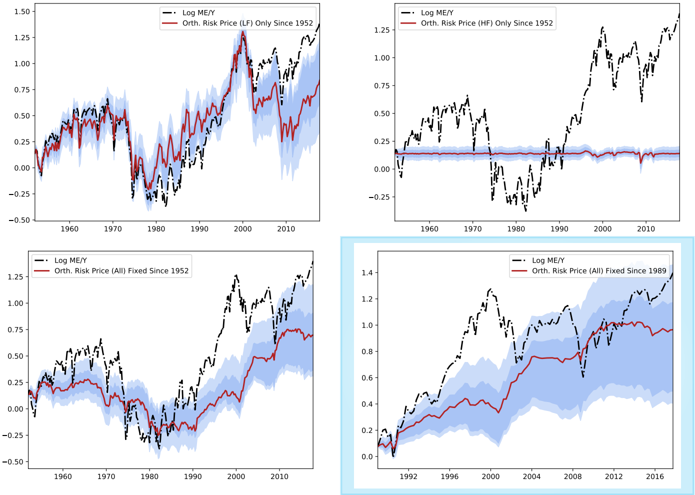
Equity Dynamics: Role of Risk Price

- Small portion of rise in py_t since 1989 explained by the decline in $x_{\perp,t}$ based on modal parameter values.



The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk-price component $x_{\perp,t}$. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Growth Decomposition

- **Market's rise**: **43%** since 1989 and 19% over full sample attributable to s_t.

	Panel: Market Equity		
Contribution	1952-2017	1952-1988	1989-2017
Total Δ ME	1405.81%	151.23%	477.34%
Factor Share	18.57%	-23.34%	42.53%
$s_{LF,t}$	17.05%	-21.59%	37.88%
$s_{HF,t}$	1.52%	-1.75%	4.64%
Risk Price $x_{\perp,t}$	25.73%	20.46%	24.42%
$x_{\perp,LF,t}$	0.05%	-0.32%	24.32%
$x_{\perp,HF,t}$	25.68%	20.78%	0.10%
Risk-Free Rate	2.16%	-8.52%	8.48%
$\delta_{LF,t}$	2.11%	-8.57%	8.35%
$\delta_{LF,t}$	0.05%	0.05%	0.13%
Real PC Output Growth	53.54%	111.41%	24.57%

The table decomposes total growth in market equity (ME) into component sources. Parts attributable to a single source are obtained by fixing all other components at their values at beginning of the sample. Component sources named in the first column sum to 100% of observed growth in ME. The sample spans the period 1952:Q1-2017:Q4.

Growth Decomposition

- **Market's rise**: **24%** since 1989 and 26% over full sample attributable to $x_{\perp,t}$.

	Panel: Market Equity		
Contribution	1952-2017	1952-1988	1989-2017
Total Δ ME	1405.81%	151.23%	477.34%
Factor Share	18.57%	-23.34%	42.53%
$s_{LF,t}$	17.05%	-21.59%	37.88%
$s_{HF,t}$	1.52%	-1.75%	4.64%
Risk Price $x_{\perp,t}$	25.73%	20.46%	24.42%
$x_{\perp,LF,t}$	0.05%	-0.32%	24.32%
$x_{\perp,HF,t}$	25.68%	20.78%	0.10%
Risk-Free Rate	2.16%	-8.52%	8.48%
$\delta_{LF,t}$	2.11%	-8.57%	8.35%
$\delta_{LF,t}$	0.05%	0.05%	0.13%
Real PC Output Growth	53.54%	111.41%	24.57%

The table decomposes total growth in market equity (ME) into component sources. Parts attributable to a single source are obtained by fixing all other components at their values at beginning of the sample. Component sources named in the first column sum to 100% of observed growth in ME. The sample spans the period 1952:Q1-2017:Q4.

Growth Decomposition

- **Much smaller role** for the risk-free rate

	Panel: Market Equity		
Contribution	1952-2017	1952-1988	1989-2017
Total Δ ME	1405.81%	151.23%	477.34%
Factor Share	18.57%	-23.34%	42.53%
$s_{LF,t}$	17.05%	-21.59%	37.88%
$s_{HF,t}$	1.52%	-1.75%	4.64%
Risk Price $x_{\perp,t}$	25.73%	20.46%	24.42%
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Growth Decomposition

- **Economic Growth** contributes **just 25%** since 1989; 54% over full 65 year sample.

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Growth Decomposition

- **1952-1988**: Δy_t explained **111%** of market's rise. But...

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Growth Decomposition

- That **37 year period** created *less than a third* of wealth generated in **29 years** from 1989 to end of 2017.

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Growth Decomposition

- Market made far greater gains in much shorter time from 1989-present, when factor shares **reallocated rewards** to equity-holders even as economic growth slowed.

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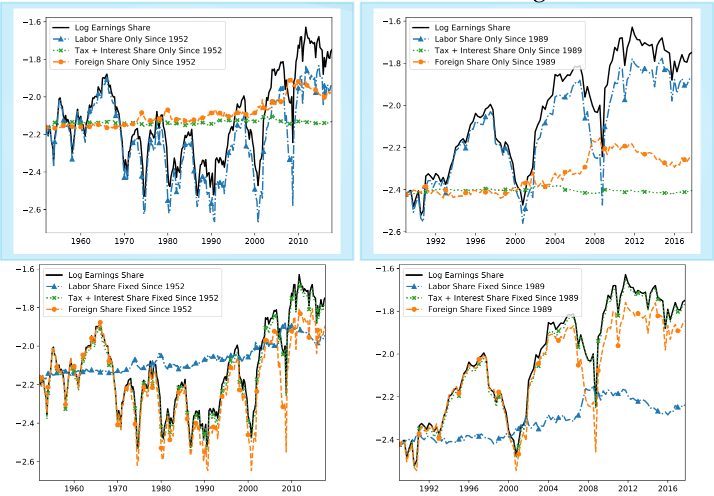
What Explains Upward Trend in the Earnings Share?

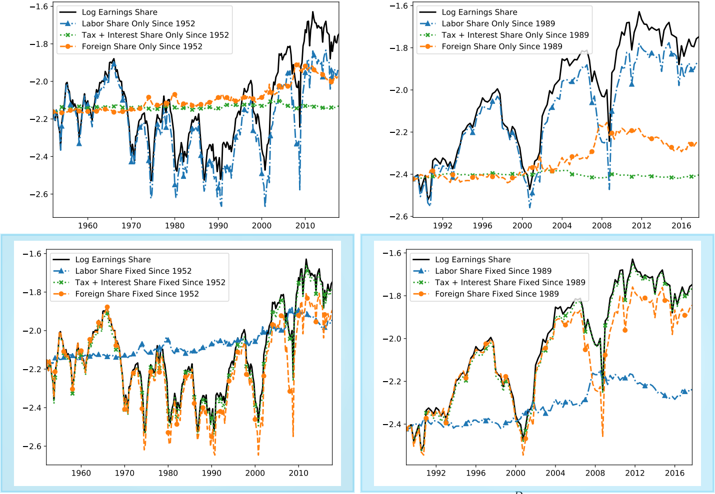
- Allow only one component to vary at a time: declining domestic labor share accounts for bulk of rise in earnings share.



The figure decomposes the corporate earnings share S_t into contributions from changes in the domestic labor share S^D, the tax and interest share Z, and the foreign share F. Series denoted "only" show the result of allowing only that component to vary, while the others are held fixed at their initial values for that period (1952 or 1989). Series denoted "Fixed" show the result of leaving that one component fixed at the start of the period while allowing all of the other components to vary. The sample spans the period 1952:Q1-2017:Q4.

What Explains Upward Trend in the Earnings Share?

- Fixing one component at a time, explain little of run-up in earnings share w/ fixed labor share.



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Summary

- **Why has the market risen** over the post-war period?
- We estimate **flexible parametric model** allows influence from several latent components, while **inferring values** components must have taken to explain the data.
- **Find** high returns to holding equity due in large part to good luck, attributable to **string of shocks that reallocated rewards** toward shareholders away from workers.
- Realizations **added 2.9 p.p.** to mean log excess return, according to estimates (overstating risk premium by $\approx 44\%$).
- **For 37 years** from 1952-1989, **economic growth drove the stock market**.
- But that period was **comparatively lackluster for equity values**, creating less than a third of the wealth generated over the 29 years from 1989 to end of 2017.

APPENDIX

Contrast with Literature

- Our results differ from contemporaneous papers such as Farhi and Gourio, 2018 and Corhay, Kung, Schmid, 2020 who find falling interest rates play crucial role in driving equity values in recent decades.
- These papers measure changes across steady states, in which parameters can change only **permanently**.
- They therefore interpret observed drop in risk-free rates as a permanent shift, leading in their models to a *huge* increase in market value.
- Since the implied increase in ME from falling $r_{f,t}$ would be even larger than the actual increase observed, these models infer that *risk-premia must have risen* at same time.

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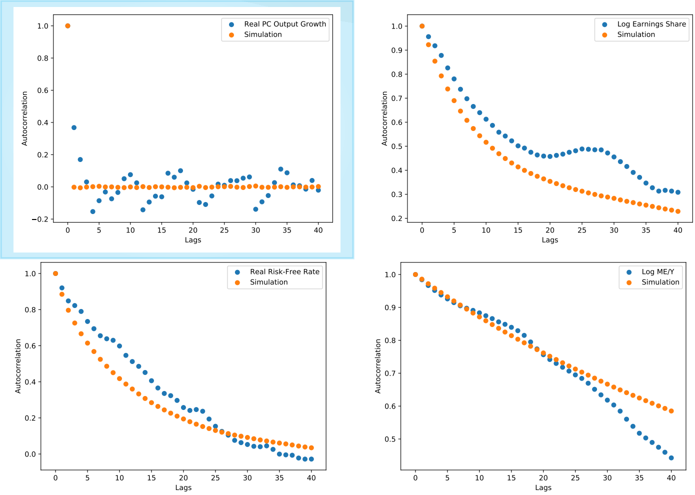
- Since the implied increase in ME from falling $r_{f,t}$ would be even larger than the actual increase observed, these models infer that *risk-premia must have risen* at same time.
- By contrast, our model views interest rate changes as far from permanent, since **we estimate** the dynamic model. We view our approach as strongly preferred by the data.
- Next figure: autocorrelations of $r_{f,t}$ deeply inconsistent with a process dominated by permanent shocks.

Observable Autocorrelations

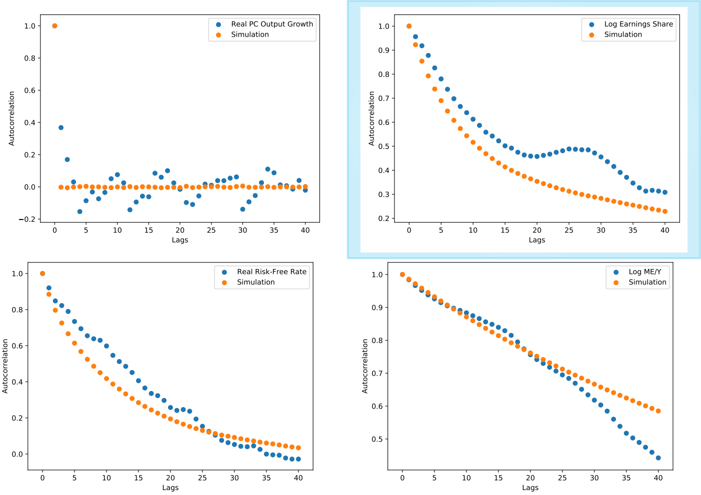
- Both model and data, autocorrelations of Δy_t hover around zero, suggesting a near i.i.d. process



The figure compares the data autocorrelations for the observable variables available over the full sample, compared to the same statistics from the model, obtained from a long simulation of 100,000 periods. The sample spans the period 1952:Q1-2017:Q4.

Observable Autocorrelations

- Autocorrelations of ey_t, $r_{f,t}$, and py_t converge to zero and suggest persistent but stationary processes.



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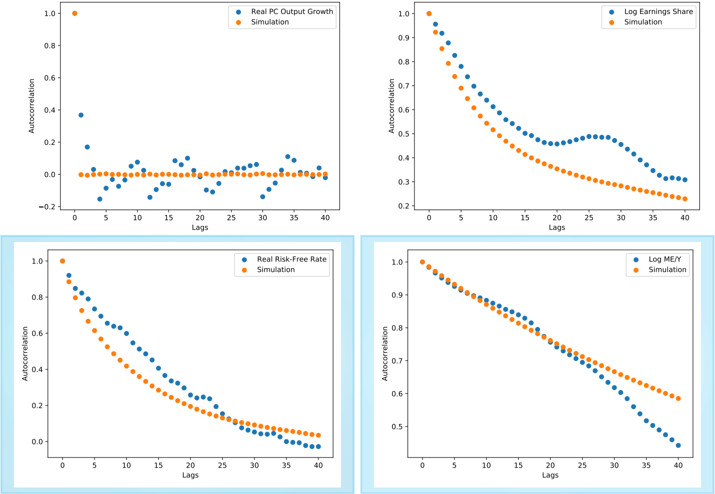
Observable Autocorrelations

- Autocorrelations of $r_{f,t} \to 0$ at quarterly lag 35, those for py_t are > 0.5 at that lag.



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Loglinear Model: Equilibrium Stock Market Values

- **Equity return**: Let P_t denote total market equity, with C_t equity payout, return on equity is

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t}.$$

- $pc_t \equiv \ln\left(\frac{P_t}{C_t}\right)$. The log return obeys the following approximate identity:

$$r_{t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1},$$

where $\kappa_1 = \exp\left(\overline{pc}\right) / \left(1 + \exp\left(\overline{pc}\right)\right)$, and $\kappa_0 = \exp\left(\overline{pc}\right) + 1 - \kappa_1 \overline{pc}$.

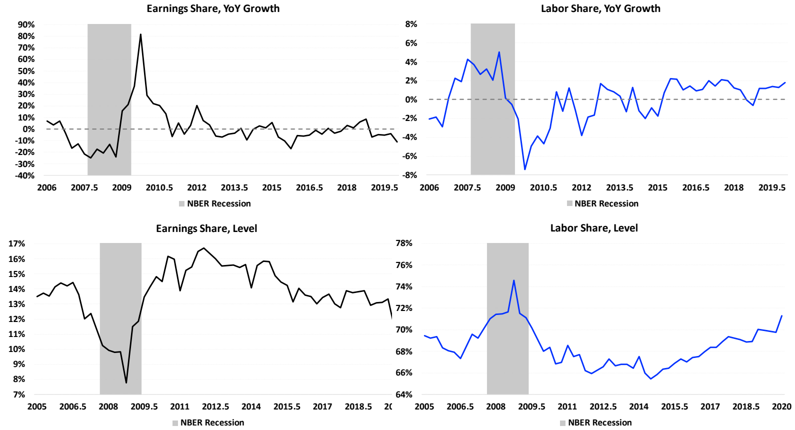
- The first-order-condition for optimal shareholder consumption:

$$\frac{P_t}{C_t} = \mathbb{E}_t \exp\left[m_{t+1} + \Delta c_{t+1} + \ln\left(\frac{P_{t+1}}{C_{t+1}} + 1\right)\right].$$

- **Conjecture and verify** a solution takes form:

$$pc_t = A_0 + \mathbf{A}_s' \widetilde{\mathbf{s}}_t + \mathbf{A}_r' \widetilde{\delta}_t + \mathbf{A}_{x_\perp}' \widetilde{\mathbf{x}}_{\perp,t}$$

Earnings and Labor shares 2005:Q1-2020:Q1



The sample spans the period 2005:Q1-2020:Q1.

Augmented model

- Augment the GLL model with a transitory component of output

$$\begin{aligned} z_{t+1} &= \phi_z z_t + \varepsilon_{z,t+1}, \qquad\qquad \varepsilon_{z,t+1} \sim N(0, \sigma_z^2) \\ \Delta z_{t+1} &= -(1-\phi_z) z_t + \varepsilon_{z,t+1}. \end{aligned}$$

- Total output growth is now defined by

$$\Delta y_{t+1} = g + \Delta z_{t+1} + \varepsilon_{a,t+1} = g - (1-\phi_z) z_t + \varepsilon_{z,t+1} + \varepsilon_{a,t+1}.$$

- Under these assumptions, the price dividend ratio is

$$pd_t = A_0 + \mathbf{A}_s' \tilde{\mathbf{s}}_t + \mathbf{A}_{x\perp}' \tilde{\mathbf{x}}_{\perp,t} + \mathbf{A}_\delta' \tilde{\delta}_t + A_z z_t \;\; \text{where} \;\; A_z = -\frac{1-\phi_z}{1-\kappa_1 \phi_z}$$

- Change in stock wealth is given by

$$\begin{aligned} \Delta p_{t+1} &= \Delta p y_{t+1} + \Delta y_{t+1} \\ &= g + H_i' \Big(-(I - \mathbf{F}) \boldsymbol{\beta}_t + \varepsilon_{t+1} \Big) - (1-\phi_z) z_t + \varepsilon_{z,t+1} + \varepsilon_{a,t+1}. \end{aligned}$$

Preliminary results

- Use initial conditions from Kalman Filter, and combine with transitory output drop implied by Survey of Professional Forecasters
  - Initial -32% decline (annualized), persistence 0.74.
- Initial conditions (-0.8%) plus output drop (-0.2%) explain little of observed drop (-33.7%).

Change (SD)	0	-1	-2	-3	-4	-5
Δs_{LF}	0.000	-0.015	-0.030	-0.045	-0.060	-0.075
Implied % ΔME	-1.0%	-3.5%	-6.0%	-8.4%	-10.8%	-13.1%
Δs_{HF}	0.000	-0.048	-0.096	-0.144	-0.192	-0.239
Implied % ΔME	-1.0%	-2.3%	-3.6%	-5.0%	-6.3%	-7.5%

Preliminary results

- Now augment with drops in profit share (Δs) of various sizes.
- Can explain larger share, but magnitudes much smaller than observed drop (-33.7%).
- Explaining full drop with FS requires a 15 Std drop in $s_{LF,t}$ or 29 SD drop in $S_{HF,t}$. The first = largest Std drop in our sample (but not post-recession); The second unheard of (largest drop 4.6 Std).

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Preliminary results

- Most likely candidate given size, speed of change and quick reversion: HF orthogonal risk price.

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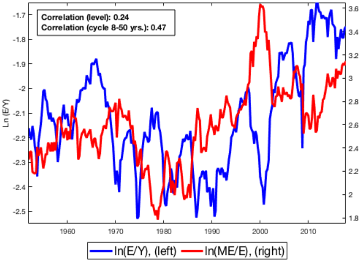
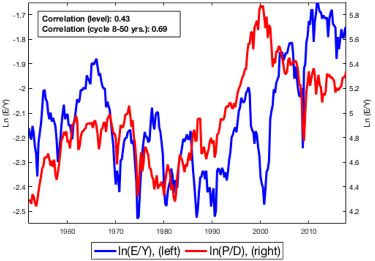
Earnings Share and Valuations





Earnings Share of Output and Equity Valuations Over the Post-War Period. ln(E/Y) denotes the logarithm of the total profit share of the U.S. corporate sector. ln(ME/E) is the log of the stock wealth-profit ratio. ln(PD) is the log of the CRSP price-dividend. Each plot present the correlation between the series (levels) and the correlation of the cycle of each series obtained using a band pass filter that isolates cycles between 8 and 50 years. The sample spans the period 1952:Q1-2017:Q4.

Back