The U.S. Public Debt Valuation Puzzle

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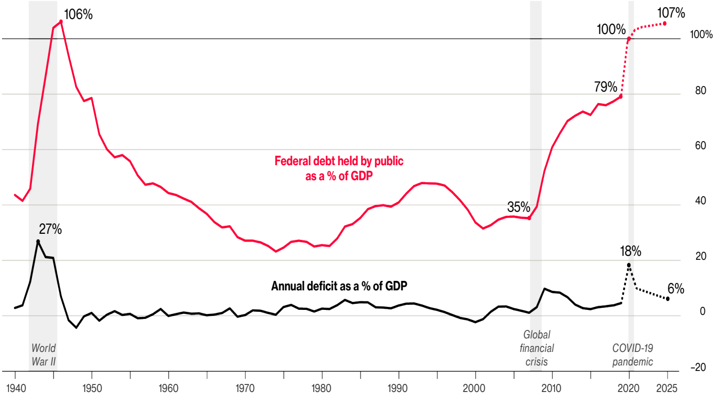
COVID-19: Largest Fiscal Shock since WWII

- estimated U.S. federal government deficit for 2020 \$4.28 trillion (about 19% of projected U.S. GDP)
    1. \$1 trillion (original deficit)
    2. \$0.6 trillion (economic changes)
    3. \$ 2.2 trillion (**+\$ 0.48 trillion**): Emergency funding: CARES Act & Families First Coronavirus Response Act
- U.S. federal government is the largest borrower in the world. The outstanding debt held by the public today is \$17.67 trillion.
- Doubled from 35% of GDP before the Great Recession to 79% in 2019. CBO/CFRB forecasts it to grow to exceed 100 % of GDP by 2021

U.S. Debt Projected to Surpass WWII record



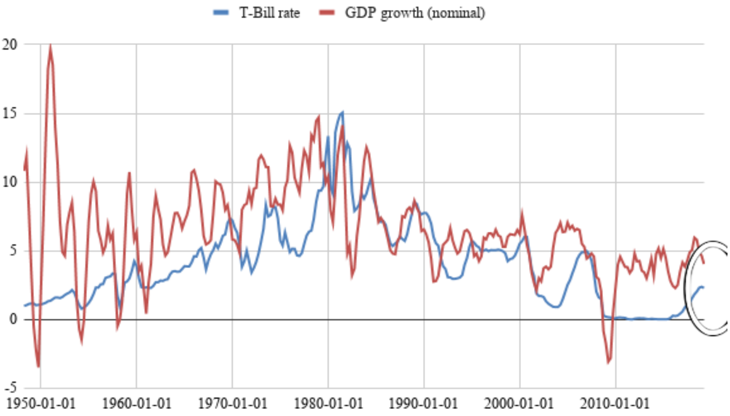
Source: Committee for a Responsible Federal Budget, Congressional Budget Office, Bloomberg data.

What is U.S.' debt-bearing capacity?

- estimates of COVID-19 crisis duration vary.
- fed gov't spending decisions seem based on 'short pause' scenario.

- Key Question for policy-makers:
  - Can the U.S. federal government easily borrow trillions more?
    - to fund the private sector's payroll,
    - to bail out states,
    - to bail out banks,
  - Or should the U.S. federal government start to reduce the deficit to avoid a debt market crash?

Let’s roll it over



- “... public debt may have no fiscal cost.” (Blanchard’s AEA Presidential address; 2019)

Government Bond Portfolio

- Government debt is backed by current and future primary surpluses.
  - Government budget constraint:

$$G_t + Q_{t-1}^1 = \sum_{h=1}^{H} \left(Q_t^h - Q_{t-1}^{h+1} \right) P_t^h + T_t$$

  - No arbitrage bond pricing: $P_t^h = \mathbb{E}_t \left[M_{t,t+1} P_{t+1}^{h-1} \right]$
  - Iterate forward on the budget constraint
  - The market value of outstanding government debt, D_t, must equal the expected PDV of future primary surpluses S_{t+j}:

$$D_t = \sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h = \mathbb{E}_t \left[\sum_{j=0}^{T} M_{t,t+j} (T_{t+j} - G_{t+j}) \right] + E_t \left[M_{t,t+T} D_{t+T} \right]$$

  - impose a TVC: $E_t \left[M_{t,t+T} D_{t+T} \right] \to 0$ as $T \to \infty$

Government Bond Portfolio

- Government debt is backed by current and future primary surpluses.
  - Government budget constraint:

$$G_t + Q^1_{t-1} = \sum_{h=1}^{H} \left(Q^h_t - Q^{h+1}_{t-1} \right) P^h_t + T_t$$

  - No arbitrage bond pricing: $P^h_t = \mathbb{E}_t \left[M_{t,t+1} P^{h-1}_{t+1} \right]$
  - Iterate forward on the budget constraint + impose a TVC

$$D_t \quad = \quad \sum_{h=0}^{H} Q^{h+1}_{t-1} P^h_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right] = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} S_{t+j} \right].$$

- Assumes existence of SDF (*not* complete markets), no arbitrage in Treasury market, no bubbles, and no convenience yield on Treasuries.

Miller-Modigliani for Treasury

- Government debt is backed by current and future primary surpluses.
  - Intertemporal Government budget constraint:

$$D_t = \sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right] = P_t^T - P_t^G.$$

  - holds ex ante in nominal and real terms
    - ex post, surprise inflation can erode real value of debt
  - allowing for default only changes left hand side, not right hand side

Treasury Balance Sheet

Assets	Liabilities
Tax Revenue P_t^T	Spending P_t^G
	Debt D_t

Return Evidence

Treasury Balance Sheet

Tax Revenue $P_t^{T,ex}$	Spending $P_t^{G,ex}$
	Debt $B = D_t - S_t$

- Risk Premium on Treasury Portfolio:

$$\mathbb{E}_t\left[R_{t+1}^D - R_t^f\right] = \frac{B_t + P_t^{G,ex}}{B_t}\mathbb{E}_t\left[R_{t+1}^T - R_t^f\right] - \frac{P_t^{G,ex}}{B_t}\mathbb{E}_t\left[R_{t+1}^G - R_t^f\right],$$

$$\beta_t^D = \frac{B_t + P_t^{G,ex}}{B_t}\beta_t^T - \frac{P_t^{G,ex}}{B_t}\beta_t^G, \beta_t^i = \frac{-cov\left(M_{t+1}, R_{t+1}^i\right)}{var_t(M_{t+1})},$$

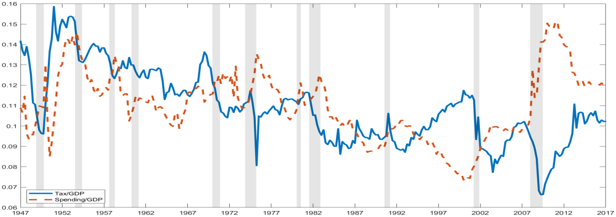
  - in U.S. data: $\mathbb{E}_t\left[R_{t+1}^D - R_t^f\right]$ is 100 bps per annum.
  - Risk-free Treasury portfolio: need safer revenue claim.

$$\beta_t^T = \frac{P_t^{G,ex}}{B_t + P_t^{G,ex}}\beta_t^G << \beta_t^G,$$

Cash Flow Evidence

$$D_t \quad = \quad \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} S_{t+j} \right] = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right]$$

- A representative investor who buys all government debt issuances and receives all redemptions has a claim to future primary surpluses $\{S_{t+j}\}$. Surpluses are the **cash flows on this investment strategy**.
- Surpluses are highly pro-cyclical: $\beta_t^T >> \beta_t^G$?



Government Debt Valuation Puzzle

- This logic poses a puzzle: Government debt is positive while surplus claim has negative value when measured in the data

$$\begin{aligned}
\underbrace{\sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h}_{>0} &= \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right] \\
&= \sum_{j=0}^{\infty} \mathbb{E}_t \left[M_{t,t+j} \right] \underbrace{\mathbb{E}_t \left[T_{t+j} - G_{t+j} \right]}_{\approx 0} \\
&+ \underbrace{\sum_{j=0}^{\infty} cov_t \left(M_{t,t+j}, T_{t+j} \right)}_{<0} - \underbrace{\sum_{j=0}^{\infty} cov_t \left(M_{t,t+j}, G_{t+j} \right)}_{>0} .
\end{aligned}$$

- For a given amount of debt that the government wants to issue, the presence of covariance terms raises the required future surpluses substantially.

Government Debt Valuation Puzzle

- This logic poses a puzzle: Government debt is positive while surplus claim has negative value when measured in the data
- The government bond portfolio is more valuable than the surplus claim:

$$\sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h > 0 > \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right]$$

- Equivalently, interest rates on the government bond portfolio in data are much lower than the risk-adjusted "interest rate" on the surplus claim. Risk premium equivalence also violated.

meas. constr.

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- Equivalently, interest rates on the government bond portfolio in data are much lower than the risk-adjusted “interest rate” on the surplus claim. Risk premium equivalence also violated.
- This puzzle is much deeper in a realistic model of risk and risk premia (SDF M);
  - most of the macro-fiscal policy literature ignores risk premia and ignores CF dynamics (except: Liu, Schmid, and Yaron, 2019)

meas. constr.

Related Literature

- **DTSM**: pricing of individual Treasury securities $P_t^h = \mathbb{E}_t\left[M_{t,t+1}P_{t+1}^{h-1}\right]$, not a portfolio of all Treasury securities;
  - new set of no-arbitrage restrictions:

$$D_t = \sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h = \mathbb{E}_t\left[\sum_{j=0}^{\infty} M_{t,t+j}(T_{t+j} - G_{t+j})\right].$$

- **optimal fiscal policy**: mostly normative work, ignore asset pricing (except: Bhandari, Evans, Golosov and Sargent, 2019)
- **fiscal sustainability**: assumes U.S. debt is risk-free and tests IBC. (except: Chernov, Schmid and Schneider, 2016; Pallara and Renne, 2019)
- **convenience yields**

Outline

- **A Few theoretical results on risk premia**
- **Value the claim to future surpluses $P_t^T - P_t^G$**
  - Specify cash flow dynamics: tax revenues T and government spending G.
    Capture both their observed cyclicality and long-run mean-reversion.
  - Specify stochastic discount factor: M.
    Able to price observed government bond and stock prices.
  - Price the claim to future tax revenues and spending: P_t^T, P_t^G.
    Quantify the magnitude of the puzzle , $D_t - \left(P_t^T - P_t^G\right)$.
- **Potential Resolutions**:
  - Convenience yield
  - Peso problem: risk of a large cut in G
  - Others: bubble, other government assets, segmented markets
- **Application**: Government maturity choice as risk management tool

Long-run Discount Rates

- Consider tax revenue strips and govt. spending strips.
- Let $R^G_{t,t+j}$ and $R^T_{t,t+j}$ be the holding period return on the j-period government spending strip and tax revenue strip.

Long-run Discount Rates

- Assume cointegration between government spending and GDP, and between tax revenues and GDP.

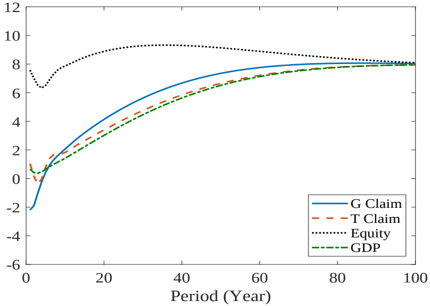
Corollary

If $\log G/GDP$ and $\log T/GDP$ are stationary in levels, then
$\lim_{j\to\infty} \mathbb{E}_t \log R^{G}_{t,t+j} = \mathbb{E}_t \log R^{GDP}_{t,t+j} >> y^{\infty}_t$, and
$\lim_{j\to\infty} \mathbb{E}_t \log R^{T}_{t,t+j} = \mathbb{E}_t \log R^{GDP}_{t,t+j} >> y^{\infty}_t$

- Long-run discount rate on G and T claims are equal to long-run discount rate on GDP, i.e., an unlevered equity claim.
  - Builds on work by Alvarez and Jermann (2005), Hansen and Scheinkman (2009), Borovicka, Hansen, and Scheinkman (2016), and Backus, Boyarchenko, and Chernov (2018).
- Latter is much higher than long-term risk-free bond yield, because of permanent shocks to marginal utility.
  - Entropy of permanent component of pricing kernel large to explain stock prices (Borovicka, Hansen, and Scheinkman, 2016)

Short-run Discount Rates

- Short-run discount rates are much higher for T-claim than for G-claim because tax revenues are pro-cyclical (like stock dividends) and government spending is counter-cyclical (recession hedge)
- Government debt investors have a net long position in a claim that is exposed to the same long-run cash flow risk as GDP and that has a lot of short-run cash-flow risk.



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- Combining short-run and long-run discount rates, AP logic predicts that tax revenue claim cannot be safer than spending claim,

$$\mathbb{E}\left[R^D_{t+1} - R^f\right] > \mathbb{E}\left[R^T_{t+1} - R^f\right] > \mathbb{E}\left[R^G_{t+1} - R^f\right]$$

- Risk premium puzzle: In the data $\mathbb{E}\left[R^D_{t+1} - R^f\right]$ is very low

Cash Flow Dynamics: Dependence on Macro Variables

- Federal govt. tax revenues $\{T\}$ are the cash-flows on the revenue claim. Federal govt. spending net of interest expenses $\{G\}$ are the cash flows on the spending claim.
- Social Security Administration aggregated with U.S. Treasury; in- and outflows into SSA budget included in T and G; intra-governmental bond holdings net out (debt held by the public).
- Define $\tau_t = \log(T_t/GDP_t)$, and $g_t = \log(G_t/GDP_t)$
- We let $\Delta\tau_{t+1}$ and Δg_{t+1} depend on lagged macro variables such as real GDP growth, inflation, interest rates, slope of YC, price-dividend ratio on stock market, aggregate dividend growth, and their own lags
- In such a model, fiscal shocks permanently affect level of τ_t and g_t; τ_t and g_t are non-stationary

Cash Flow Dynamics: Cointegration with GDP

- Additionally impose that tax revenue and spending are cointegrated with GDP
  - Johansen test indicates presence of two cointegration relationships
  - Cointegration indicates (long-run) automatic stabilizers
    - When tax revenue is above its long-run trend with GDP, revenue adjusts downwards (and spending upwards)
    - When spending is above its long-run trend with GDP, spending adjusts downwards (and tax revenue upwards)
    - Consistent with Bohn (1998)
  - Now fiscal shocks temporarily affect level of τ_t and g_t
- $\Delta\tau_{t+1}$ and Δg_{t+1} also depend on lagged cointegration variables τ_t and g_t.

Cash Flow Dynamics: Summary

- De-meaned state vector z follows VAR

$$\begin{aligned} \boldsymbol{z}_t &= [\pi_t - \pi_0, x_t - x_0, y_t^{\$}(1) - y_0^{\$}(1), yspr_t^{\$} - yspr_0^{\$}(1), pd_t - \overline{pd}, \Delta d_t - \mu_d, \\ &\quad \Delta\tau_t - \mu_0^{\tau}, \Delta g_t - \mu_0^{g}, \tau_t - \tau_0, g_t - g_0]'. \end{aligned}$$

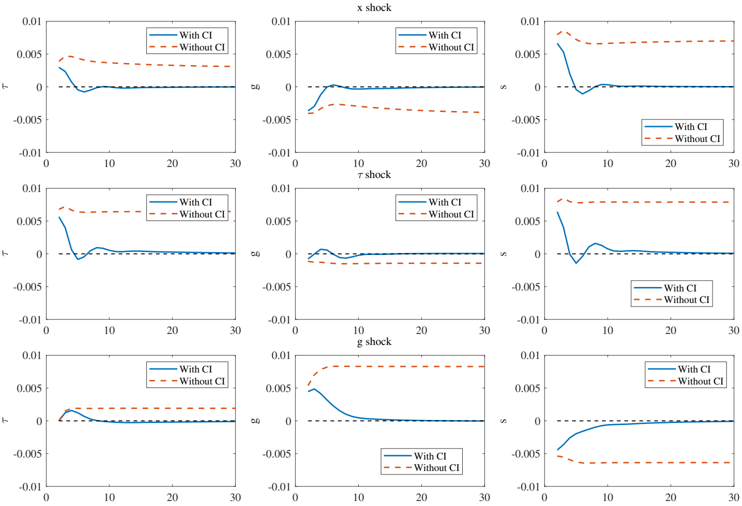
- Stationarity of $\log \tau$ and $\log g$ requires $\mu_0^{\tau} = 0$ and $\mu_0^{g} = 0$
- Post-war data implies average primary surplus of almost exactly zero
- State vector follows a VAR:

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Psi} \boldsymbol{z}_t + \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\varepsilon}_{t+1}$$

- Shocks $\boldsymbol{\varepsilon}_{t+1} \sim \mathcal{N}(0, I)$. Cholesky decomposition of residual covariance matrix $\Sigma = \Sigma^{\frac{1}{2}'} \Sigma^{\frac{1}{2}}$.
- Estimate state variable dynamics by OLS on 1947.Q1-2017.Q4: $\widehat{\Psi}$, $\widehat{\Sigma}$

▸ VAR

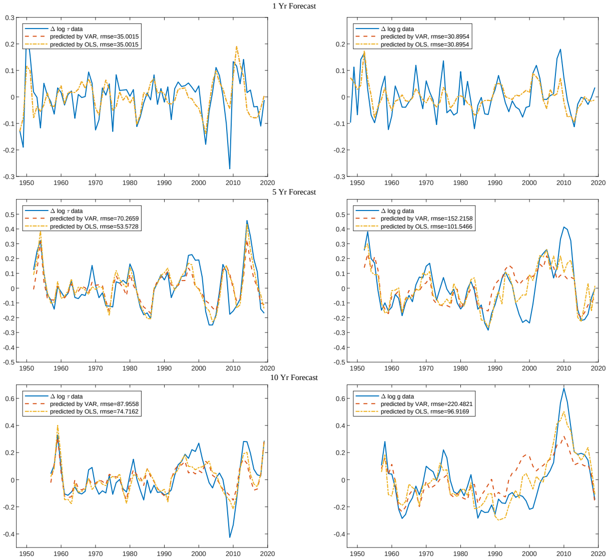
Responses of Tax and Spending



- Also verify that quarterly VAR implies realistic *long-horizon* predictability of govt spending and tax revenue growth by τ_t and g_t

details

Forecasts


1 Yr Forecast
Δ log τ data
predicted by VAR, rmse=35.0015
predicted by OLS, rmse=35.0015
Δ log g data
predicted by VAR, rmse=30.8954
predicted by OLS, rmse=30.8954
5 Yr Forecast
Δ log τ data
predicted by VAR, rmse=70.2659
predicted by OLS, rmse=53.5728
Δ log g data
predicted by VAR, rmse=152.2158
predicted by OLS, rmse=101.5466
10 Yr Forecast
Δ log τ data
predicted by VAR, rmse=87.9558
predicted by OLS, rmse=74.7162
Δ log g data
predicted by VAR, rmse=220.4821
predicted by OLS, rmse=96.9169

Asset Pricing Model

- Takes a stance on the priced sources of aggregate risk in the economy
  - Inflation and real GDP growth
  - Level & slope factor in the bond term structure
    - 4-factor model for term structure of government bond yields
  - Dividend growth on the stock market
- Affine log SDF with market prices of risk Λ_t (Ang and Piazzesi, 2003)

$$\begin{aligned} m^{\$}_{t+1} &= -y^{\$}_t(1) - \frac{1}{2}\mathbf{\Lambda}'_t\mathbf{\Lambda}_t - \mathbf{\Lambda}'_t\boldsymbol{\varepsilon}_{t+1} \\ \mathbf{\Lambda}_t &= \mathbf{\Lambda}_0 + \mathbf{\Lambda}_1 z_t \end{aligned}$$

- Bond yields, price-dividend ratios on stock strips, expected (excess) returns on bonds and stocks are all affine in z_t

MPR

Pricing Claims to Revenue T and Spending G

- With VAR dynamics and the SDF in hand, we can value T and G claims

$$
\begin{aligned}
P_t^T &= \mathbb{E}_t\left[\sum_{j=0}^{\infty} M_{t,t+j} T_{t+j}\right] \\
P_t^G &= \mathbb{E}_t\left[\sum_{j=0}^{\infty} M_{t,t+j} G_{t+j}\right].
\end{aligned}
$$

- The price-dividend ratios $PD_t^T = P_t^T / T_t$ and $PD_t^G = P_t^G / G_t$ are affine in the state z_t.
- Value of the surplus claim is $P_t^T - P_t^G = T_t PD_t^T - G_t PD_t^G$
- Scale by GDP for easier comparison to debt/GDP

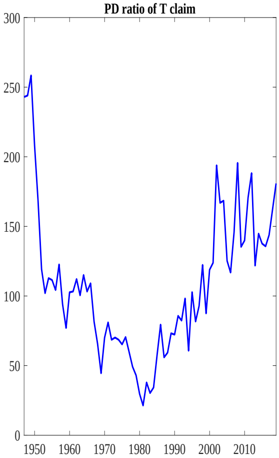
$$
\frac{T_t}{GDP_t} PD_t^T - \frac{G_t}{GDP} PD_t^G
$$

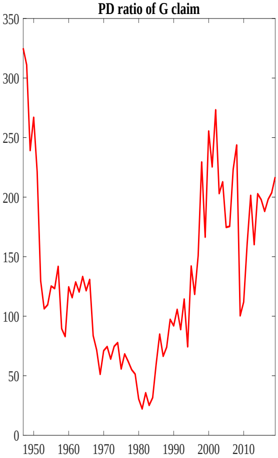
Main Results

- Now consider the main model with cointegration
- Log revenue/GDP and log spending/GDP are stationary

Main Results

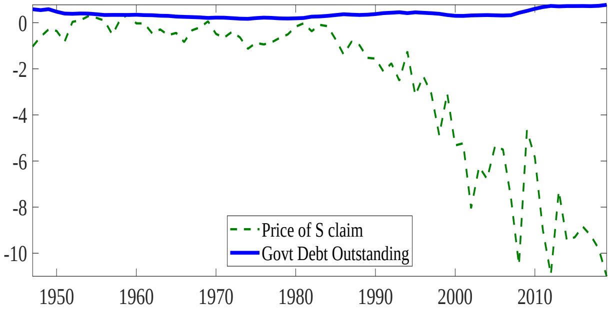
- Valuation ratio T-claim is 108.65, annual risk premium of 6.17% per year
- Valuation ratio G-claim is 133.52, annual risk premium of 6.62% per year





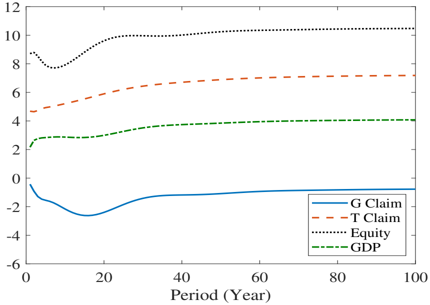
Main Results

- $P_t^\tau - P_t^g$ is value of surplus claim; averages to -2.57 $\times$ GDP
- Should equal the market value of debt; average to +0.37 $\times$ GDP
- Government debt capacity has come down in last decade, while outstanding debt has doubled: wedge has grown *fivefold* in 20 years



Main Results

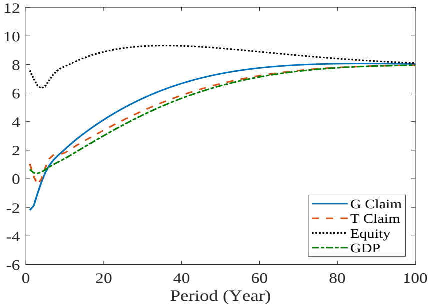
- Term structure of discount rates without cointegration



  - Without cointegration, permanent shocks to T and G lead to very high long-run risk premia on T claim vs. G claim

Main Results

- Term structure of discount rates with cointegration



- Without cointegration, permanent shocks to T and G lead to very high long-run risk premia on T claim vs. G claim
- With cointegration, long-run expected return on T- and on G-claim equals long-run expected return on GDP claim; higher than long bond yield, b/c permanent component in SDF

Potential Resolutions: Convenience Yield

- We define convenience yield λ_t as Treasury bonds paying lower yields than implied from SDF:

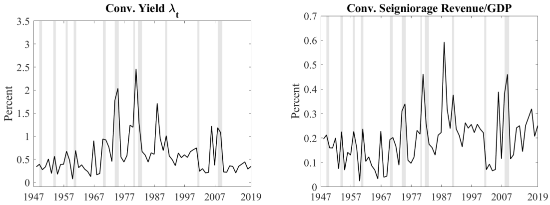
$$\begin{aligned} E_t[M_{t+1}] &= P_t^1 e^{-\lambda_t}, \\ E_t[M_{t+1}P_{t+1}^1] &= P_t^2 e^{-\lambda_t}, \\ E_t[M_{t+1}P_{t+1}^K] &= P_t^{K+1} e^{-\lambda_t}. \end{aligned}$$

- Debt not only backed by EPDV of surpluses but also by revenue from convenience services that Treasury debt offers investors:

$$D_t = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(T_{t+j} - G_{t+j} + (1 - e^{-\lambda_{t+j}})D_{t+j}\right)\right]$$

Convenience Yield in the Data

- Measure λ_t as the weighted average of CP-TBill spread and AAA-TBond spread (Krishnamurty and Vissing-Jorgensen, 2012).
  - Average is 60bps p.a.; seigniorage revenue averages 0.20% of GDP



How Large a Convenience Yield to Close the Gap?

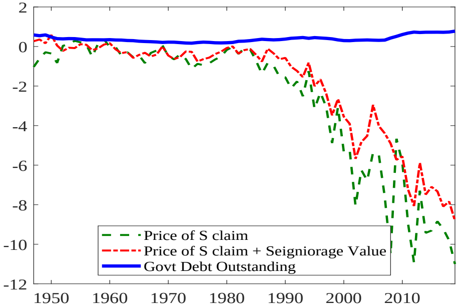
- The convenience yield can be thought of as seignorage revenue

$$D_t = E_t \left[\sum_{j=0}^{\infty} M_{t,t+j} \left(\underbrace{T_{t+j} + (1 - e^{-\lambda_{t+j}}) D_{t+j}}_{T_{t+j} K_{t+j}} - G_{t+j} \right) \right]$$

- Add $\Delta \log K_{t+1}$ to the VAR, estimate its dynamics along with other state variables in z_{t+1}
- Add λ_t to all the Treasury yields to get to "safe rates" without convenience yield

How Large a Convenience Yield to Close the Gap?

- PDV of convenience yield seigniorage worth 14.5% of GDP on average
- Reduces puzzle but does not resolve it.

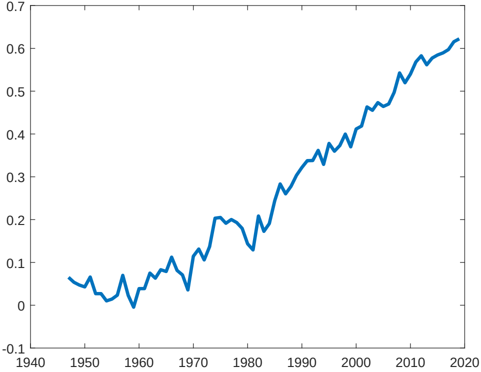


Potential Resolutions: Peso Problem

- Hypothesize that probability ϕ_t of a significant, permanent spending cut is priced in the government debt
- But such a spending cut "disaster" never realizes in post-war U.S. era
- We set the government spending cut to 8% of U.S. GDP = 2$\times$stdev of spending shock. Average spending is 11.5% of GDP in sample.
- How large should this spending cut probability ϕ_t be in order to equate the market value of the government debt to the present value of surpluses, period-by-period?

Spending Cut

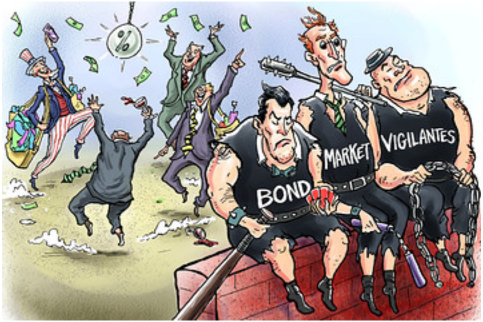
- Large implied probability suggests a restatement of the puzzle:



Bubble in Treasurys

- Bubble $=$ value of outstanding debt $-$ value of surplus claim
- Our approach quantifies the size of the bubble at 300% of GDP unconditionally
- bond markets are not enforcing TVC
- limits to arbitrage

Where have all the bond market vigilantes gone?



Contributing Factors

- convenience yields are much larger and more counter-cyclical than you think
  - U.S. is world's designated supplier of dollar-denominated safe assets (*but that could change*)
  - other dollar-denominated assets (not Treasurys) also earn convenience yield
  - Krishnamurthy, Jiang and Lustig (2019): much larger convenience yields for foreigners (between 2 and 3%)
- Fed is distorting yields

Revisiting Blanchard

- the right discount rate $R^d > g$ is much higher than g
  - maybe the return on Treasurys satisfies $R^d - \lambda < g$, but λ depends on supply of dollar-denominated safe assets (see Krishnamurthy and Vissing-Jorgensen, 2012)
  - if we keep issuing IOUs, at some point $R^d > g$
- and the US will have to implement pro-cyclical fiscal policy to lower R^d (as do many other countries)

- U.K. was world's safe asset supplier until WWI
- U.K. lost its status as supplier of safe assets after WWI
- defaulted on its 5% WWI bond, converted in 3.5% bonds in 1932
  'If you cannot fight, you can help your country by investing all you can in 5 percent Exchequer bonds. Unlike the soldier, the investor runs no risk'

Conclusion

- pricing all Treasurys: A strategy that buys all outstanding U.S. Treasurys produces risky cash flows. It should carry a high risk premium.
  - Requires appropriately quantifying sources of aggregate risk that are reflected in bond prices and stock prices, not just fiscal shocks like in macro literature.
- Implies that bond yields are puzzlingly low, especially recently.
- conventional estimates of convenience yield cannot close the gap, meaning:
  1. investors hold optimistic beliefs about future fiscal rectitude (especially after COVID-19 shock)
  2. investors fail to enforce the TVC.
  3. convenience yields are much larger than we think.