Growth-Indexed Securities
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Once upon a time

The Author

- *Displacement Risk and Asset Returns* (Gârleanu, Kogan, Panageas; JFE, 2012)
- Aggregate consumption and aggregate dividend cointegrated?
  - ▸ Is the value of the stock market a claim to consumption stream
  - ▸ Market incompleteness: stock market does not include the value yet-to-be invented ventures

Other People

- Sargent-Wallace, Sims, etc...
- Government Debt is a claim on fiscal surpluses

$$D = \mathrm{P}(\text{taxes}) - \mathrm{P}(\text{expenditures})$$

- What if $\mathrm{P}(\text{expenditures}) \to +\infty$ ...
- ... do not worry taxes and expenditures are *cointegrated*

Other People

- By the way: $r < g$

This paper

Cointegrated has a specific meaning

- If C and X are cointegrated, there is a linear combination of both that is *stationary*
- A process is stationary if its distribution is invariant over time

Finance

- Laws of statistics in finance usually translate but not necessarily
- Stationarity under $\mathbf{P}$ measure does not imply stationarity under a different measure $\mathbf{Q}$
- What if that other measure is the pricing measure?

Quickly

Simple cointegration

- Simplest consumption process: constant $c_t = C_0$
- Dividend process is mean-reverting $dx_t = -\kappa(x_t - x_0)dt + \sigma dW$
- Define stochastic process as the ratio

$$z_t = \frac{x_t}{c_t} = \frac{1}{c_0}x_t$$

  - z_t is stationary!

Pricing measure

- Take some pricing measure $\mathbf{Q}$ such that R-N derivative introduces some constant drift
  - $d\mathbf{Q}/d\mathbf{P} = \exp\left(\int_0^t as dW_s - \frac{1}{2}\int_0^t (as)^2 ds\right)$
- Change in measure introduces a constant drift ... z_t is not stationary under $\mathbf{Q}$

$$dz_t = -\kappa\left(z_t - \frac{x_0}{c_0}\right)dt + \sigma\frac{at}{c_0}dt + \sigma d\hat{W}_t$$

Complete version

- Things move around: c_t is stochastic, so are dividends x_t
- Moreover x_t depends on c_t (dividend policy is state contingent)
- Under a general dividend distribution rule
  - Wealth consumption ratio is stationary
  - So is the dividend to wealth ratio
- There is a dividend policy that is cointegrated with consumption

Complete version

The dividend claim

- Price of strips go to 0
  - ▸ The value of all the strips is bounded: it is the value of the asset

$$\mathrm{P}_t(X_T) \xrightarrow[T \to +\infty]{} 0$$

The consumption claim

- Standard pricing with a fixed risk-premium

$$\mathrm{P}_t\left(\frac{C_T}{C_t}\right) = \exp\left(-(r + \kappa\sigma - g)(T - t)\right)$$

- Not implausible case ... $r + \kappa\sigma < g$

$$\mathrm{P}_t(C_T) \xrightarrow[T \to +\infty]{} +\infty$$

Complete version

What happened to cointegration of C and X?

$$\mathrm{P}_t(C_T) \xrightarrow[T \to +\infty]{} +\infty; \qquad \mathrm{P}_t(X_T) \xrightarrow[T \to +\infty]{} 0$$

- Pricing operator is a change of measure: distorts the probabilities
- The ratio $\zeta = \log(X/C)$ is stationary
- The ratio of prices is not $\mathfrak{p}(\zeta) = \mathrm{P}(X)/\mathrm{P}(C)$

- Actual proof is slightly more complicated
- It is constructive: build a portfolio strategy

Who cares or who should care?

Asset pricing in a world where $r < g_c$

- Under some conditions, this is actually ok

Sustainability of debt

- Jiang, Lustig, Van Nieuwerburgh, and Xiaolan present value equality

$$D = \mathrm{P}(\text{taxes}) - \mathrm{P}(\text{expenditures})$$

  - ‣ Maybe prices of taxes and expenditures are not cointegrated
  - ‣ The price of a claim to the difference does not goes to zero at large horizon
  - ‣ Bubble, specialness of U.S. treasury ...

- Matters for the sustainability of debt trajectory given path of fiscal surpluses

Some practical applications

Actual GDP-indexed bonds

- France 1956-1975: coupon is $\bar{r} + \alpha\Delta_{0,t}$GDP
- Useful to evaluate whether $r > g$
    - Back out risk-adjusted growth $g^{\mathbf{Q}}$ from prices and risk-premium

$$\underbrace{g^{\mathbf{Q}}}_{2.6\%} > \underbrace{r}_{2.1\%}$$

    - Suggesting ideas around bubbles and transversality conditions are not completely outside of the empirical realm
- Usual caveat on the specific and novel bonds used to estimate $g^{\mathbf{Q}}$
    - Liquidity, size, risk, market participants

Should government issue GDP-linked bonds

- What could go wrong?
- Debt payments are now a function of GDP
    - Automatic stabilizer: no more drift in debt/gdp ratio (if sole portfolio)
- If there is a "bubble" component in the price of government bonds
    - Isn't is optimal to use it?

- What shape would monetary policy take in a world of GDP-indexed bonds?

Final Thoughts

Very interesting Paper!

Take away

- Clean explanation of some confusion in the literature
- Flesh out the difference between cointegration and the pricing of two processes

Great Paper!